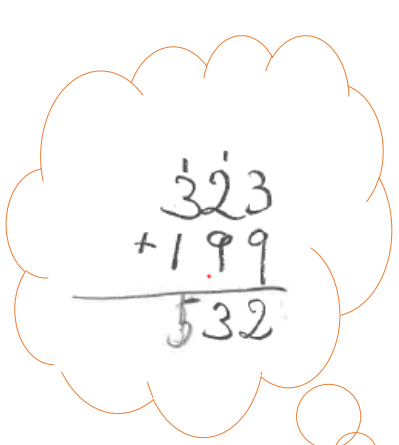
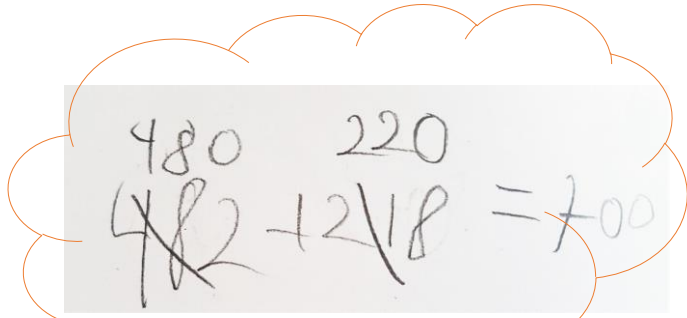
Strategibrug og de udfordrede elever
Eksempler fra dansk forskning


Pernille Bødtker Sunde


Lóa Björk Jóelsdóttir





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TRYGFONDENS BØRNEFORSKNINGSCENTER

Hvad ved vi om strategier og udvikling i matematik

Tidlig strategibrug og senere læring

Strategibrug i starten af 1. klasse kan forudsige senere matematiske færdigheder og kompetencer i bl.a. brøker, ligninger og tekstopgaver.

Strategier er en bedre predictor end en standardiseret matematikscore (fx MAT-test).

Strategier er udtryk for bagvedliggende tal- og regneforståelse.

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Check for updates

Grade one single-digit addition strategies as predictors of grade four achievement in mathematics

Pernille Bødtker Sunde[1,2] · Bert De Smedt[3] · Lieven Verschaffel[2] · Peter Sunde[4]

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Abstract
Early detection of and relevant information on children's mathematical difficulties is important to initiate targeted teaching and intervention. This study investigated the extent to which strategy use in single-digit addition provides additional predictive information about 61 grade one children's (6-year-old) mathematical achievement 3 years later that is not available from a standardised mathematics achievement test. Four predictors available in year one (arithmetic strategy use, mathematical achievement, non-verbal reasoning skills and sex) explained 54% of the variation in grade four mathematics achievement. Arithmetic strategy use was the most important single predictor of year four mathematics achievement ($R^2 = 30\%$) and explained an additional 12% variation if added to a model comprised by the three other year one predictors. This result suggests that systematically obtained measures of how young children solve single-digit arithmetic problems might provide useful information about their foundational number knowledge, which in turn may reveal how well they achieve later in school.

Vi ved også

- At udvikling af strategifleksibilitet hænger sammen med viden om tal og relationer mellem tal (McMullen et al., 2016) og konceptual og procedural viden (Rittle-Johnson et al., 2012; Schneider et al., 2011).
- At vanskeligheder i matematik ofte er relateret til vanskeligheder med etcifret regning (Funderud et al., 2019; Mazzocco et al., 2008; Vanbinst et al., 2014).
- At de lavt præsterende anvender mere ‘umodne’ strategier og er mere afhængige af back-up strategier (Ostad, 1999; Siegler, 1987; Verschaffel, et al., 2007).

Talbaserede metoder er mere sikre end standardalgoritmen

- Standardalgoritmen er den hyppigst anvendte
- Selvom de talbaserede (adaptive) strategier er mest sikre
- Gælder både addition og subtraktion

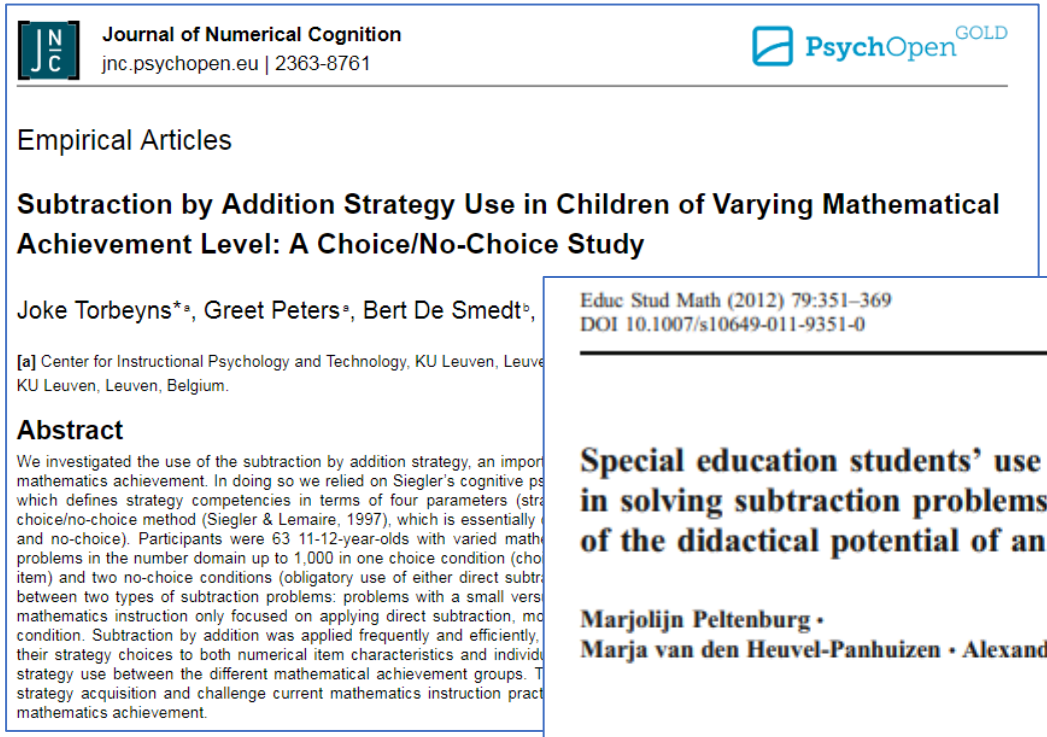
Tabel 6. *Strategier anvendt til subtraktion (693 – 499 eller 673 – 199) fordelt på klassetrin.*

	3. klasse		6. klasse		8. klasse	
	% af løste opgaver	% rigtige	% af løste opgaver	% rigtige	% af løste opgaver	% rigtige
Standardalgoritme	33 %	37 %	61 %	72 %	81 %	80 %
Dekomposition (split)	5 %	25 %	2 %	59 %	1 %	91 %
Sekventielle strategier (træk fra)	8 %	46 %	4 %	68 %	2 %	67 %
Sekventielle strategier (tæl op)	2 %	33 %	8 %	60 %	4 %	65 %
Kompensation	1 %	100 %	6 %	95 %	3 %	96 %
Ingen beskrevet metode	22 %	11 %	8 %	45 %	4 %	28 %
Fejlforståelse	19 %		7 %		3 %	
Tæller	1 %	0 %	0 %		0 %	
Andet	10 %	18 %	5 %	61 %	2 %	72 %

Jóelsdóttir, L. B. & Sunde, P. (2024).

Alle elevgrupper kan anvende talbaserede metoder med succes

– også udfordrede elever!



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Empirical Articles

Subtraction by Addition Strategy Use in Children of Varying Mathematical Achievement Level: A Choice/No-Choice Study

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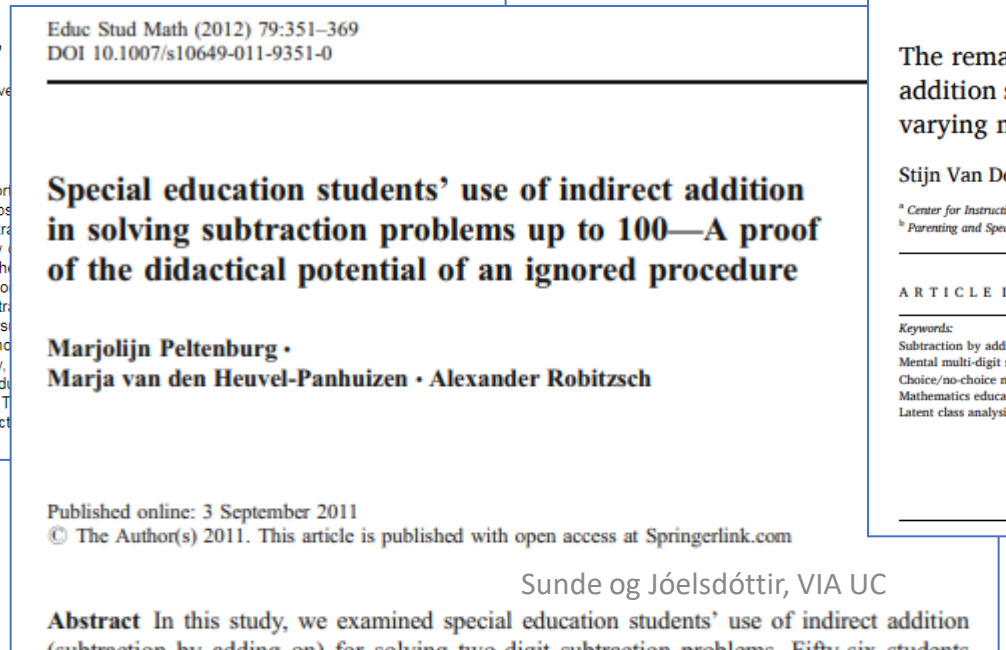
[a] Center for Instructional Psychology and Technology, KU Leuven, Leuve
KU Leuven, Leuven, Belgium.

Abstract

We investigated the use of the subtraction by addition strategy, an impor
mathematics achievement. In doing so we relied on Siegler's cognitive ps
which defines strategy competencies in terms of four parameters (stra
choice/no-choice method (Siegler & Lemaire, 1997), which is essentially
and no-choice). Participants were 63 11-12-year-olds with varied math
problems in the number domain up to 1,000 in one choice condition (cho
item) and two no-choice conditions (obligatory use of either direct subtr
between two types of subtraction problems: problems with a small vers
mathematics instruction only focused on applying direct subtraction, mo
condition. Subtraction by addition was applied frequently and efficiently,
their strategy choices to both numerical item characteristics and individ
strategy use between the different mathematical achievement groups. T
strategy acquisition and challenge current mathematics instruction pract
mathematics achievement.



Educ Stud Math (2012) 79:351–369
DOI 10.1007/s10649-011-9351-0

Special education students' use of indirect addition in solving subtraction problems up to 100—A proof of the didactical potential of an ignored procedure

Marjolijn Peltenburg · Marja van den Heuvel-Panhuizen · Alexander Robitzsch

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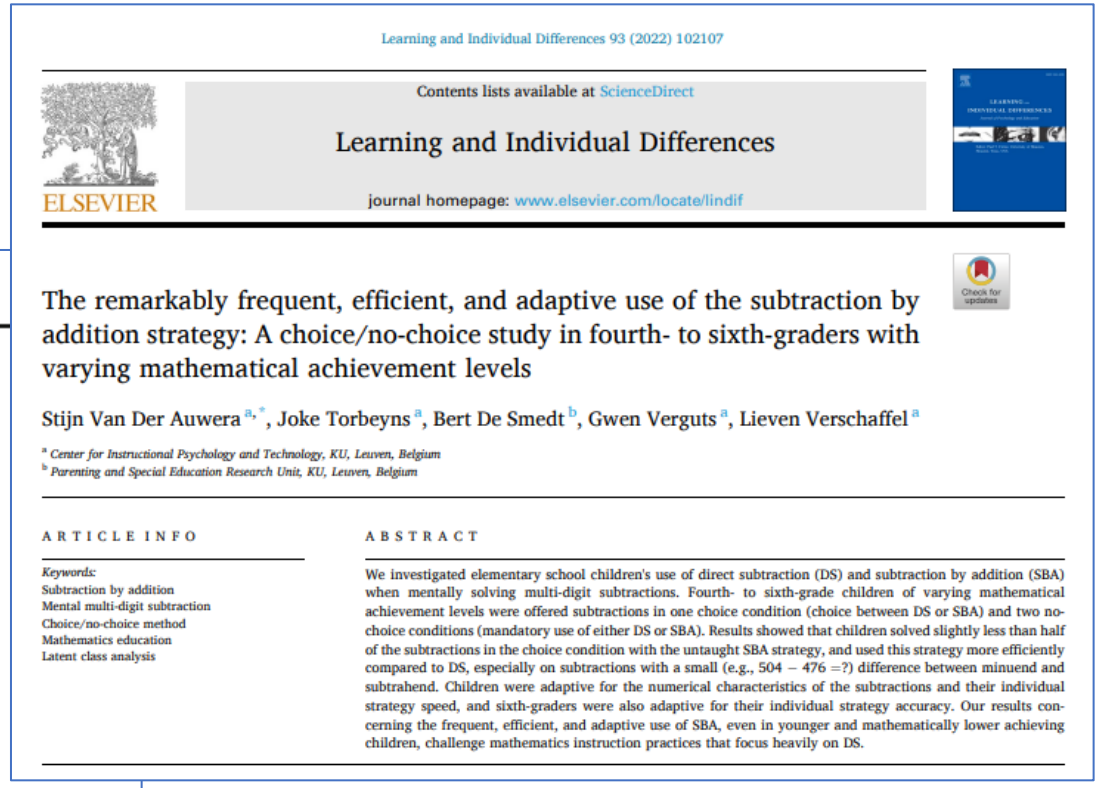
Abstract In this study, we examined special education students' use of indirect addition



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ELSEVIER

The remarkably frequent, efficient, and adaptive use of the subtraction by addition strategy: A choice/no-choice study in fourth- to sixth-graders with varying mathematical achievement levels

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ABSTRACT

We investigated elementary school children's use of direct subtraction (DS) and subtraction by addition (SBA) when mentally solving multi-digit subtractions. Fourth- to sixth-grade children of varying mathematical achievement levels were offered subtractions in one choice condition (choice between DS or SBA) and two no-choice conditions (mandatory use of either DS or SBA). Results showed that children solved slightly less than half of the subtractions in the choice condition with the untaught SBA strategy, and used this strategy more efficiently compared to DS, especially on subtractions with a small (e.g., 504 − 476 =?) difference between minuend and subtrahend. Children were adaptive for the numerical characteristics of the subtractions and their individual strategy speed, and sixth-graders were also adaptive for their individual strategy accuracy. Our results concerning the frequent, efficient, and adaptive use of SBA, even in younger and mathematically lower achieving children, challenge mathematics instruction practices that focus heavily on DS.

Subtraktion som indirekte addition

– *En mulig succes for de udfordrede elever*

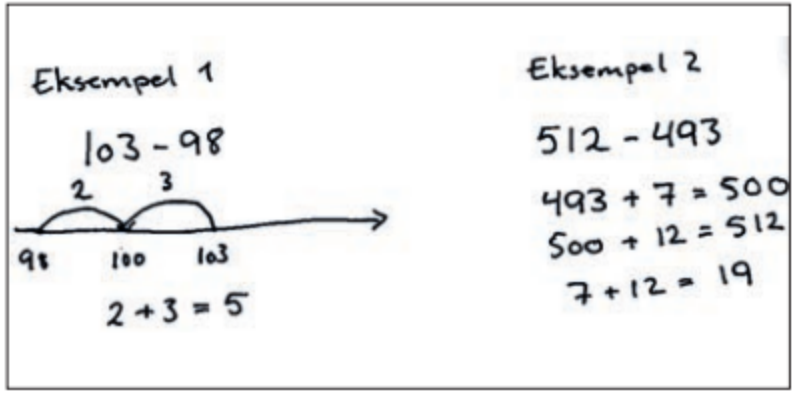
Lóa Björk Jóelsdóttir

Pernille Bødtker Sunde

Af Lóa Björk Jóelsdóttir, *lektor, VIA University College, loho@via.dk*
og Pernille Bødtker Sunder, *forsker, VIA University College, pbos@via.dk*

Subtraktion med flercifrede tal er en særlig udfordring for mange elever. Det viser både resultater fra de afsluttende prøver i matematik og international forskning. I denne artikel ser vi på indirekte addition som en regnestrategi, som har vist sig at være en effektiv strategi, også for de udfordrede elever i matematikundervisningen.

MATEMATIK
Nr. 3 · 2024



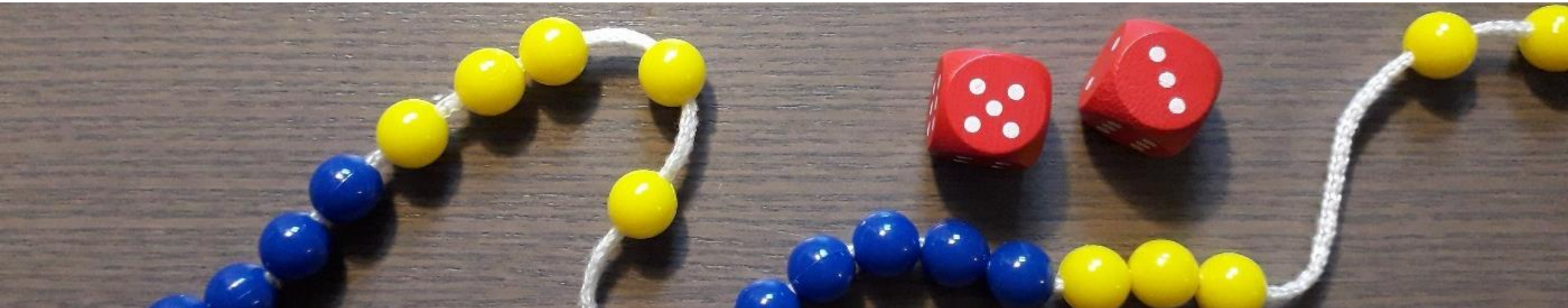
Figur 2 Eksempler på indirekte addition i subtraktion med lille forskel

Strategier og fleksibilitet er anbefalet i curriculum

- Strategier og fleksibilitet er inkluderet i læreplanerne i lande som USA, Australien, Singapore (Rittle-Johnson et al., 2012), Holland (Hickendorff, 2018), Flandern (Torbeyns et al., 2018) og Danmark (Ministeriet for Børn og Undervisning, 2019).
- I den danske læreplan, Fælles mål, står:

  " *Det er centralt, at læreren udfordrer og støtter de enkelte elever på en måde, så eleverne udvikler deres regnestrategier på baggrund af deres talforståelse frem for at lære procedurer for opstilling og udregning. Der sigtes ikke mod opøvelsen af standardiserede algoritmer..*" (Læseplanen s. 15 (min understregning))
- Men det er ikke altid anbefalet for alle elever, som fx i USA og Holland, hvor elever i matematikvanskeligheder anbefales kun at blive undervist i én metode (Peltenburg et al., 2012).
- Der mangler systematisk forskning om hvorvidt flexibilitet er gavnligt for de lavt præsterende elever (Verschaffel, Torbeyns, De Smedt, et al., 2007).

Hvilke strategier bruger de udfordrede elever?
Og er det de bruger godt for dem?



Hvad gør de udfordrede elever?

Derfor her vi undersøgt:

- Brug af talbaserede strategier og standardalgoritmen i flercifret regning
- Graden af fleksibilitet
- 'Rigtighed' (altså om eleven regner korrekt)
- Sammenhængen til niveau i matematik (Nationale test)

Regnemetoder hos elever i 6. klasse

Deltagere

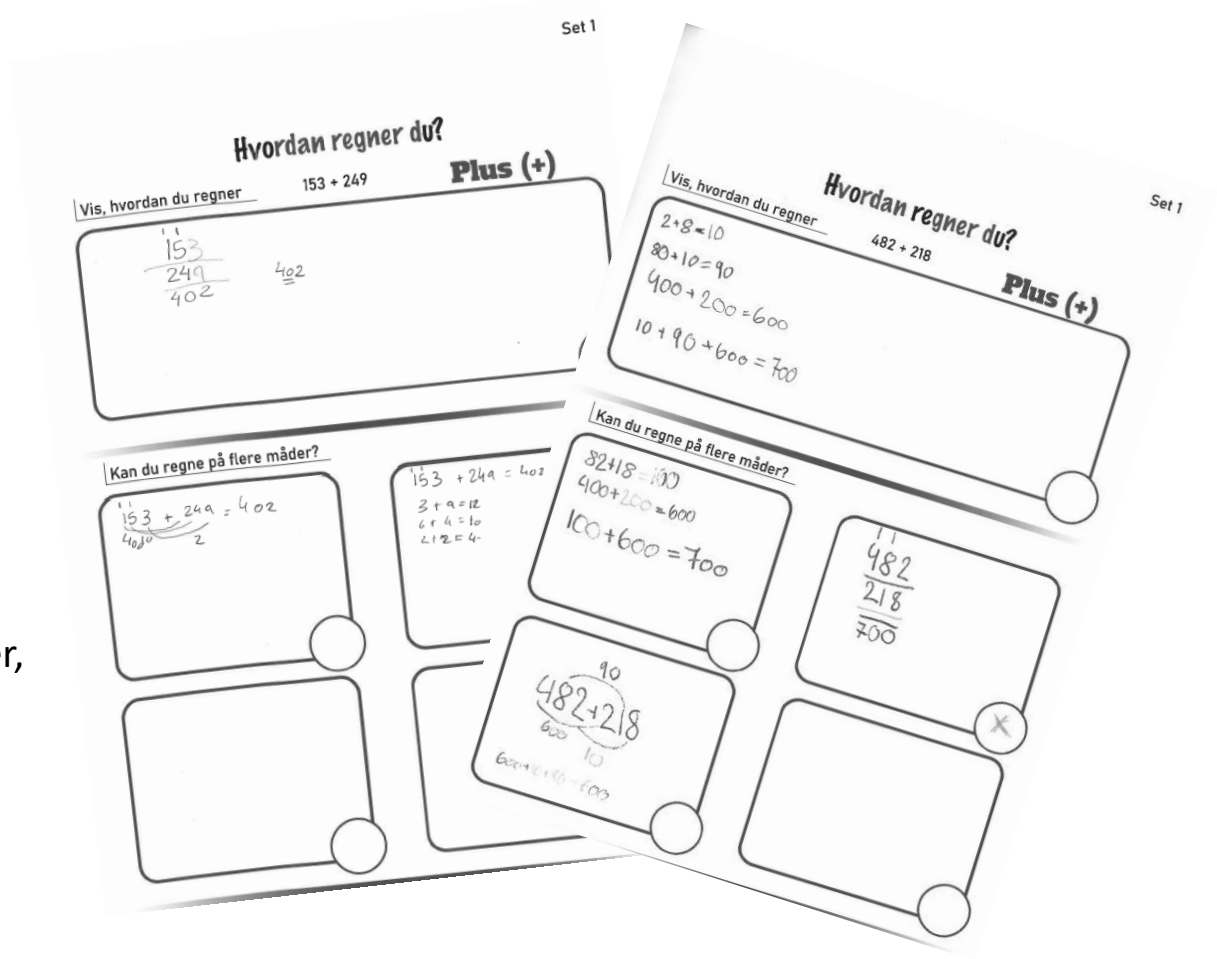
- 685 6. klasses elever
- 39 klasser – 19 skoler – 5 kommuner
- Mean age 12.5 (0.36)

Data:

Strategibrug: Tri-phase Flexibility Assessment (TriFA)

- Fase 1
  - Elevernes foretrukne metode for 9 flercifrede regnestykker, 3 addition, 3 subtraktion and 3 multiplikation
- Fase 2
  - Elevernes viden om og evne til at anvende forskellige metoder til den samme opgave - fleksibilitet

National test – 6. klasse



Kodning

Strategierne er kodet som

- (i) talbaserede strategier,
- (ii) standardalgoritmen,
- (iii) ingen løsning (intet forsøg på at løse opgaven) eller
- (iv) andet.

- Accuracy: korrekte svar i fase 1.
- Flexibility: mindst en ny strategi i fase 2. Evalueres for hver af de 9 opgaver.

- Alle besvarelser blev dobbeltkodet og i tilfælde af uenighed blev en tredje ekspert konsulteret

Talbaserede strategier			Standard algoritme
Addition 323 + 199			
300 + 100 = 400 20 + 90 = 110 3 + 9 = 12 400 + 110 + 12 = 522 300 + 100 = 400 23 + 99 = 122 400 + 122 = 522	323 + 100 = 423 423 + 90 = 513 513 + 9 = 522	323 + 200 = 523 523 − 1 = 522 323 − 1 + 199 + 1 322 + 200 = 522	1 1 323 + 199 522
Subtraktion 103 – 98			
103 − 100 = 3 3 + 2 = 5 103 + 2 − 98 + 2 105 − 100 = 5	103 − 90 = 13 13 − 8 = 5	100 − 90 = 10 3 − 8 = 5 10 − 5 = 5 98 + 2 = 100 100 + 3 = 103 2 + 3 = 5	9 0 10 13 1 0 3 − 98 5
Multiplikation 12 x 15			
12 · 15 = 6 · 30 = 180 12 · 10 = 120 120 : 2 = 60 120 + 60 = 180	10 · 10 = 100 2 · 10 = 20 10 · 5 = 50 2 · 5 = 10 100 + 20 + 50 + 10 = 180	12 · 10 = 120 12 · 5 = 60 120 + 60 = 180 15 · 10 = 150 15 · 2 = 30 150 + 30 = 180	1 12 · 15 30 + 15 180

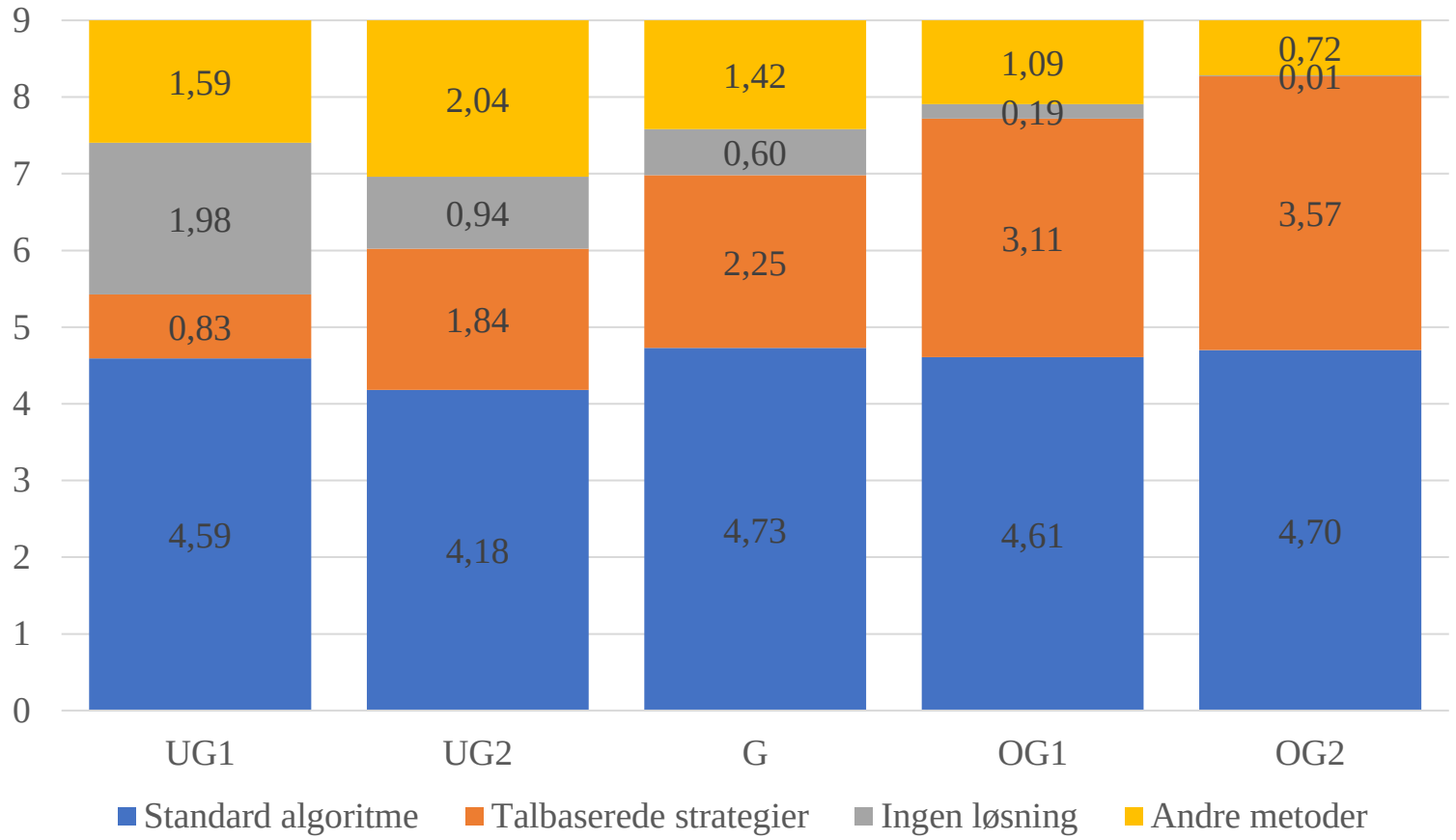
Præstationsniveauer i nationale test – fordelingen af elever

Opdeling i 5 præstationsniveauer i dansk national test i matematik, som defineret af undervisningsministeriet i den normbaserede kategorisering (UVM, 2017) og fordeling af de deltagende elever på de 5 niveauer.

	Lavest præsterende		Normalt præsterende	Højest præsterende	
	En del under gennemsnittet (UG1)	Under gennemsnittet (UG2)	Gennemsnittet (G)	Over gennemsnittet (OG1)	Langt over gennemsnittet (OG2)
Andel elever i gruppen	10 %	25%	30%	25%	10%
Normbaseret fordeling	[0-10%]	]10%-35%]	]35%-65%]	]65%-90%]	]90%-100%]
N	54	143	202	191	95
Andel deltagende elever (%)	8	21	29	28	14

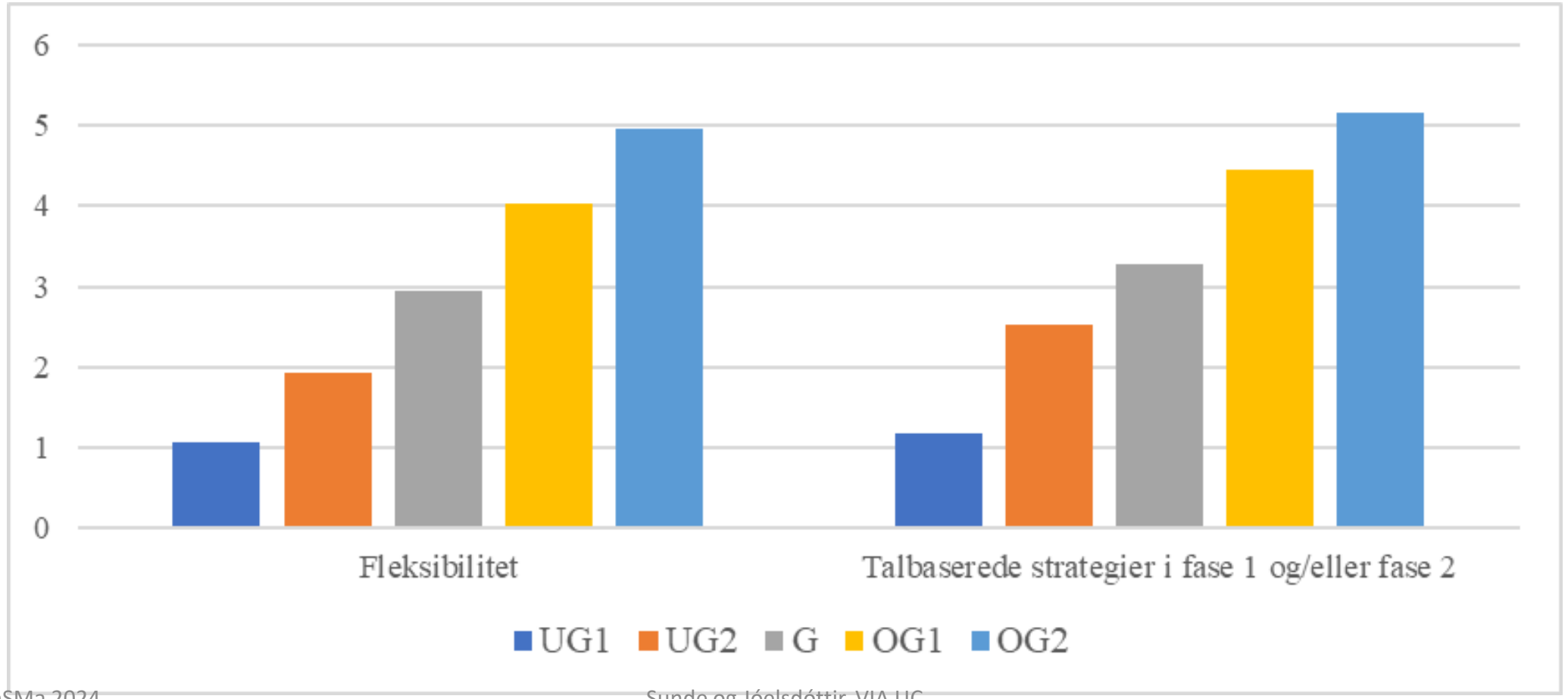
Strategibrug (means)fordelt på niveau i nationale test

Gennemsnit for antal løste opgaver i fase ét, med talbaserede strategier, standard algoritme, ingen løsningsmetode og andre metoder som foretrukne strategityper i fase ét, opdelt efter præstationsniveauer i matematik



Flexibilitet og talbaserede strategier, fordelt på niveau i nationale test

Gennemsnit score for fleksibilitet og brug af løste opgaver med talbaserede strategier i fase ét og/eller fase to. Det maksimale antal opgaver er ni. (Min 0; Max 9).



Accuracy – rigtighed – fordelt på niveau i nationale test

Rigtighed for talbaserede strategier og standard algoritme, målt for hvert niveau.

	Talbaserede strategier	**Standard algoritme**			
Niveau	Gennemsnit (*SD*)	Gennemsnit (*SD*)	[95% CI]	*t*	*p*
En del under gennemsnit (UG1)	0,53 (0,40)	0,67 (0,26)	-0.14 [-0,30, 0,03]	-1,64	0,11
Under gennemsnit (UG2)	0,80 (0,30)	0,73 (0,25)	0,07 [-0,01, 0,15]	1,81	0,07
Gennemsnit (G)	0,84 (0,26)	0,79 (0,23)	0,05 [-0,01, 0,11]	1,74	0,08
Over gennemsnit (UG1)	0,88 (0,22)	0,84 (0,22)	0,03 [-0,02, 0,08]	1,32	0,19
En del over gennemsnit (UG2)	0,93 (0,17)	0,87 (0,19)	0,05 [-0,01, 0,11]	1,79	0,08
I alt	0,84 (0,26)	0,80 (0,24)	0,05 [0,02, 0,08]	3,10	**<0,00**

Konklusion

Stadig kun korrelative sammenhænge!
Hvad med kausaliteten?

- Alle elevgrupper anvender overvejende standardalgoritmen
- Brug af standardalgoritmen er ikke associeret med matematikniveau
- De lavt præsterende viser meget begrænset brug af talbaserede strategier
- Standardalgoritmen har lavere 'rigtighed' sammenlignet med talbaserede metoder, hvilket indikerer at alle elevgrupper vil have glæde af et øget focus på de talbaserede metoder.

Mere forskning!

Det næste projekt:

Design and evaluation of an intervention to improve adaptivity in multidigit arithmetic

- Samarbejde med Trygfondens Børnforskningscenter og Professor Helena Skytt Nielsen, AU
- Støttet af DFF: 3,16 mill.

- Kan adaptivitet i flercifret regning påvirkes ved en intervention?
- Fører øget adaptivitet i flercifret regning til en øgning i generelle matematikkompetencer?

Fortsættelse følger….. i 2027

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