

# Singapore Maths and the Struggling Learner



Judy Hornigold

# Contents

- Why learners struggle with maths
- Different forms of dyscalculia
- Singapore maths
- Other strategies to help

# Why can maths be so hard?

Cumulative

Perceptions

Performance focus

Timed

Mental maths

Too much content

Too abstract

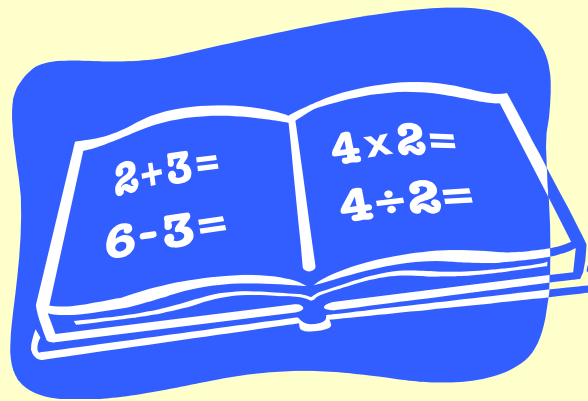


# What are the different forms of dyscalculia?

- Acalculia
- General difficulties with maths
- Dyscalculia
- Pseudo-dyscalculia

# What is dyscalculia?

- How is it different from being bad at maths?



# Definitions of Dyscalculia

- Dyscalculia is a condition that affects the ability to acquire arithmetical skills. (DfES 2001)
- A congenital condition: its effects on the learning of numerical skills can be very profound. (Butterworth)
- Dysfunction in the reception, comprehension or production of quantitative and spatial information (Sharma).

# The National Numeracy Strategy DfES (2001)

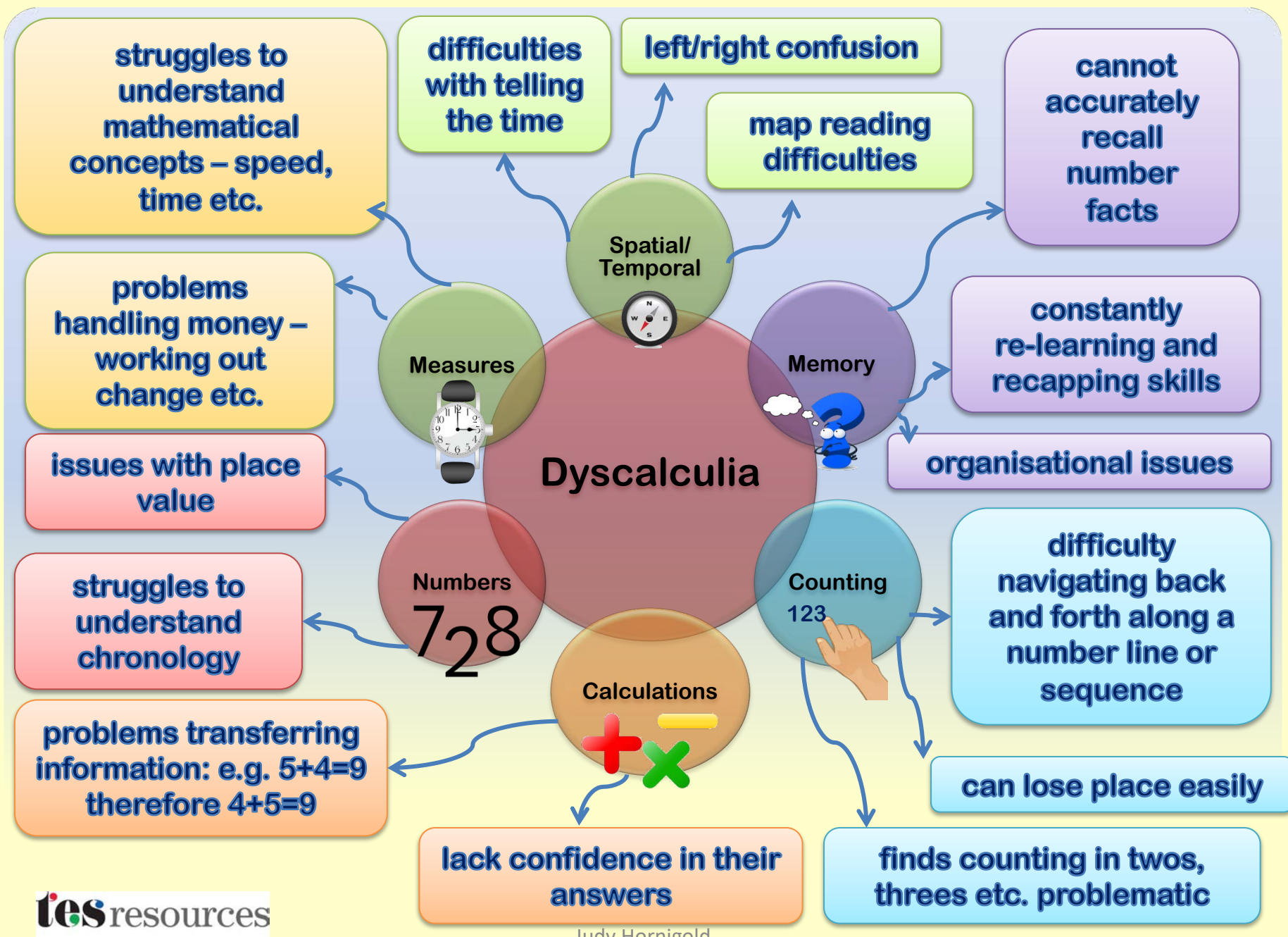
Dyscalculia is a condition that affects the ability to **acquire** arithmetical skills. Dyscalculic learners may have **difficulty understanding simple number concepts, lack an intuitive grasp of numbers,** and have problems **learning number facts and procedures.** Even if they produce a correct answer or use a correct method, they may do so mechanically and **without confidence**

# DSM-IV (2000)

## Mathematics Disorder:

"as measured by a standardised test that is given individually, the person's **mathematical ability is substantially less than would be expected** from the person's age, intelligence and education. This deficiency materially **impedes academic achievement or daily living**"





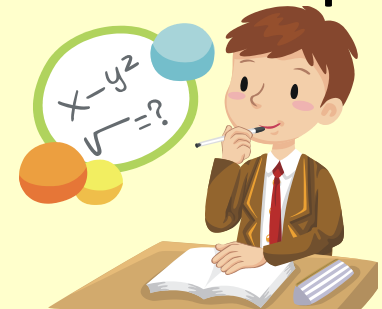
# Indicators of Dyscalculia

- An inability to subitise even very small quantities
- Poor number sense
- An inability to estimate whether a numerical answer is reasonable
- Immature strategies- for example counting all instead of counting on
- Inability to notice patterns
- Inability to generalise



# Indicators of Dyscalculia (cont'd)

- Slow processing speed
- Difficulty sequencing
- Difficulty with language
- Poor memory for facts and procedures
- Difficulties in word problems and multi step calculations



# How do you identify and assess for dyscalculia?

Dyscalculia Screener- Butterworth

Dynamo Profiler

DyscalculiUM- FE/HE screener

Questionnaire/Checklist

Observation

# DYSCALCULIA SCREENER(Nfer/Nelson 2003)

- Developed by Brian Butterworth
- Based on research that showed dyscalculic pupils performed worse on certain numerical processing skills

Tests include:

- Dot counting
- Number comparison
- Timed arithmetic

<http://www.gl-assessment.co.uk/products/dyscalculia-screener>

# Checklists

## BDA Dyscalculia Checklist



# Observation

In class

Observe how they attempt a question

Look for signs of stress

Encourage the learner to verbalise how they are attempting the maths

# A learned response


Anxiety rises when faced with mathematics and can be so severe that even the sight of a maths problem can lead to paralysing anxiety

Fight or flight reaction





# Physical Symptoms of Maths Anxiety

- queasy stomach, *butterflies* 
- clammy hands and feet
- increased or irregular heartbeat
- muscle tension, clenched fists
- tight shoulders

# Physical Symptoms of Maths Anxiety

- feeling faint, shortness of breath
- headache
- shakiness
- dry mouth
- cold sweat, excessive perspiration



# Psychological Symptoms of Maths Anxiety

- negative self-talk
- panic or fear
- worry and apprehension
- desire to flee the situation or avoid it altogether
- a feeling of helplessness or inability to cope



# Maths Anxiety can be related to

## **Life Experiences**

- attitudes of parents, teachers or other people in the learning environment
- some specific incident in a student's math history which was frightening or embarrassing
- poor self-concept caused by past history of failure

# Maths Anxiety can be related to

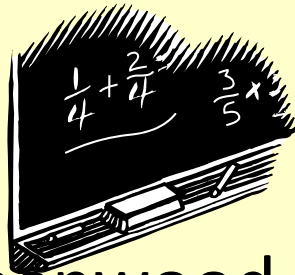
## Teaching Techniques

Anxiety can be caused by teaching techniques which emphasize:

- timed activities
- the ***right*** answer
- speed in getting the answer
- competition among students
- working in isolation
- memorization rather than understanding

# Maths lessons

- *“Evidence suggests that maths anxiety results more from the way the subject is presented than from the subject itself.”*



Greenwood (1984)

J.



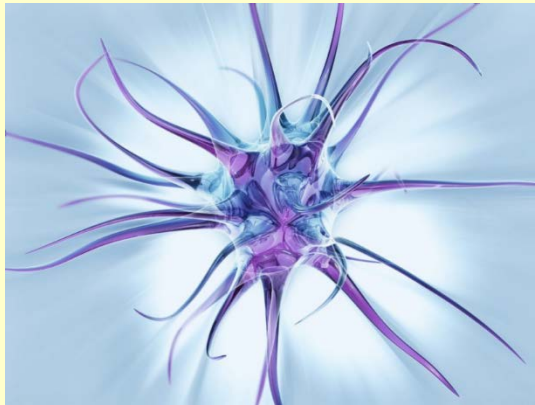
# Ways teachers can help

- Foster the idea that mistakes are good
- Spend time developing number sense at an early age
- Encourage a relaxed participatory atmosphere
- Use concrete manipulatives in a way that develops understanding

# Mistakes

- ‘Every time a student makes a mistake in maths they grow a new synapse’

Carol Dweck





# Mistakes matter

- Mistakes are vital in maths
- New data has shown that the brain grows when a mistake is made
- No growth when the answer is correct

# Mistakes are good

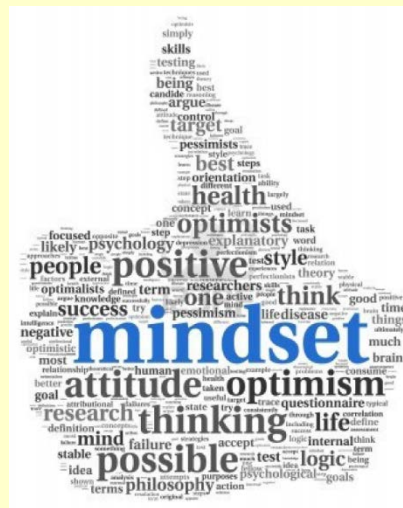
- Move from a performance culture to one where tasks are open and promote growth
- Give challenging work
- Value persistence

# Implications

- Each child has enormous potential to grow their brain, no matter where their starting point is
- Teachers have the power to take children to high levels
- Brain difference at birth has minimal impact on future learning- it is nurture rather than nature that matters.

# Mindset

How important are the ideas that students hold about their own ability?



# Carol Dweck: Mindset

- Fixed mindset: Maths ability is a gift
- Growth mindset: Maths ability grows with experience , persistence, learn from mistakes, determination to keep going, encouraged by other's success

Mindset: The New Psychology of Success (2007)

# Mindset and Maths

- Lowest achieving children are those who use a memorization technique for maths
- Highest achievers are those who look for connections and have big ideas

# Does it matter how quickly we can answer?



- Research evidence shows that maths should never be associated with speed
- Timed tests cause the early onset of maths anxiety for about  $\frac{1}{3}$  of the children in the class
- Being fast at maths is not the same as being good at maths
- Most mathematicians think slowly and deeply

# How to help

- Singapore Maths
- Problems and Solutions
- Using dot patterns
- Sharma's levels
- Key facts and derived facts

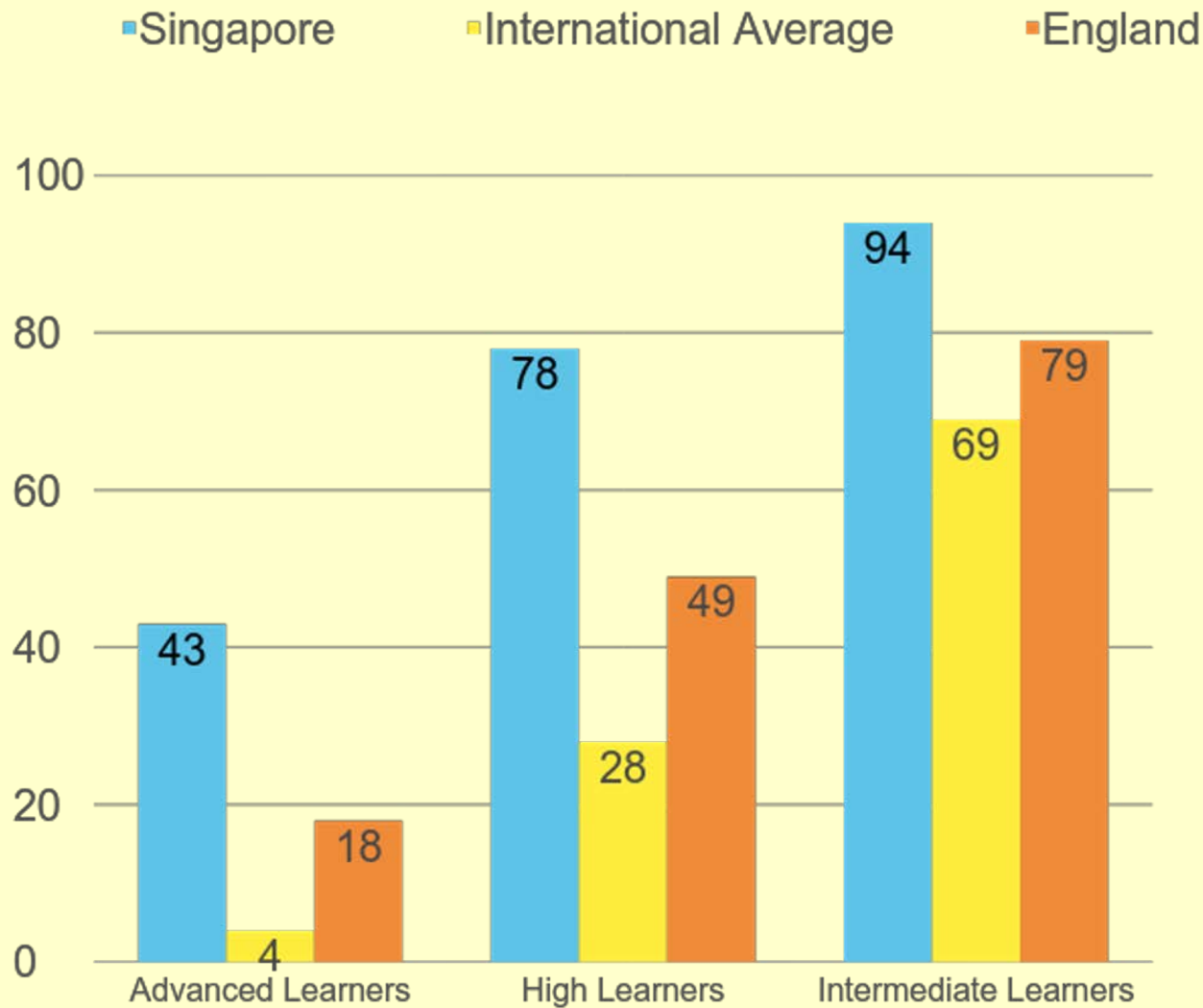


# Singapore Rises to the Top

	1995	2003	2007	2011
Singapore	590	594	599	606
Republic of Korea	581	-	-	605
Hong Kong SAR	557	575	607	602
Chinese Taipei	-	564	576	591
Japan	567	565	568	585
Russian Federation	-	532	544	542
England	484	531	541	542
United States	518	518	529	541
TIMSS Scale Average	500	500	500	500

Trends in International Math and Science Study (TIMSS)

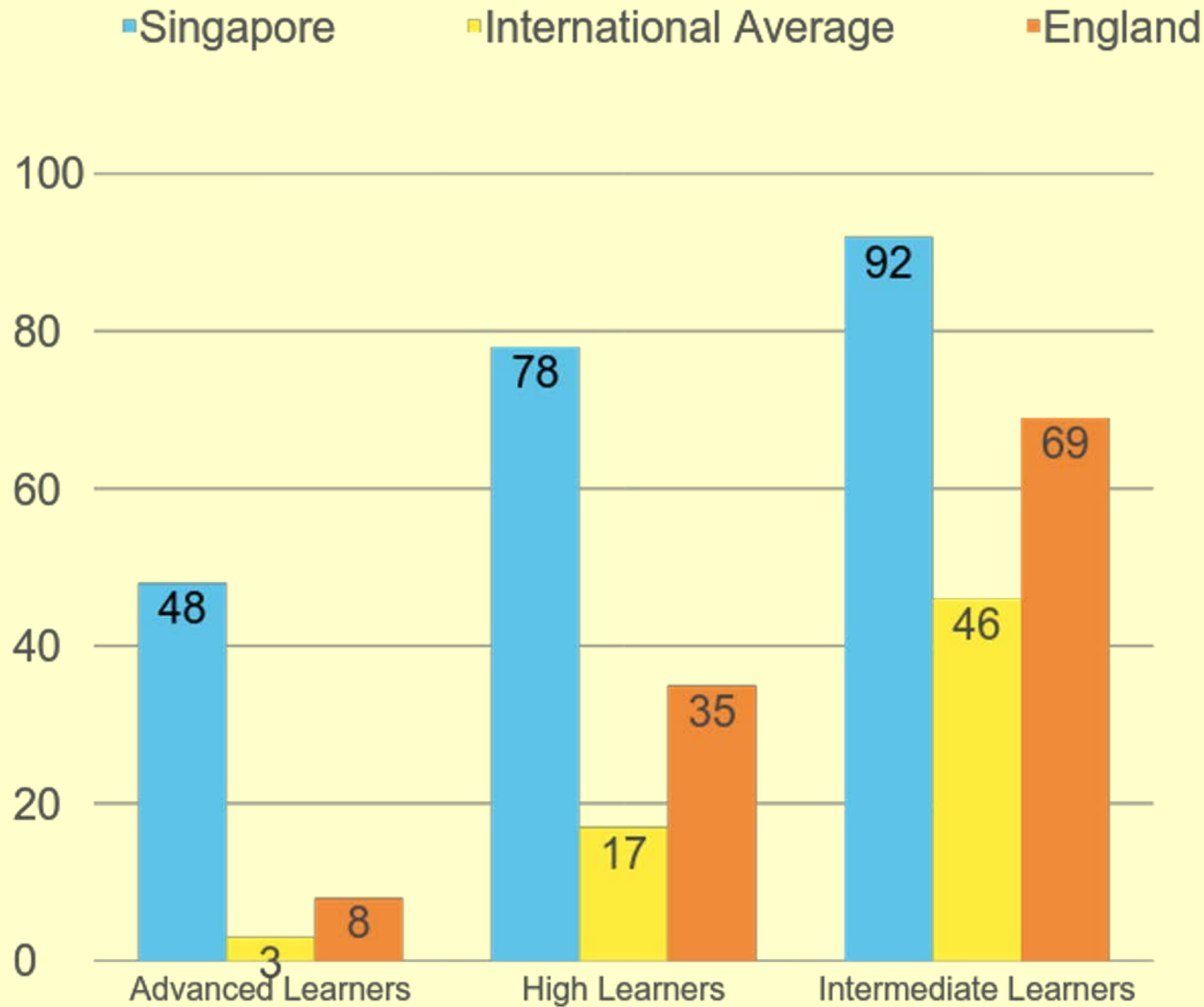
# TIMMS Benchmark 2011



*Grade 4 is the same age as UK Year 5*

*Since 1995 Singapore has been at the top of mathematics education*

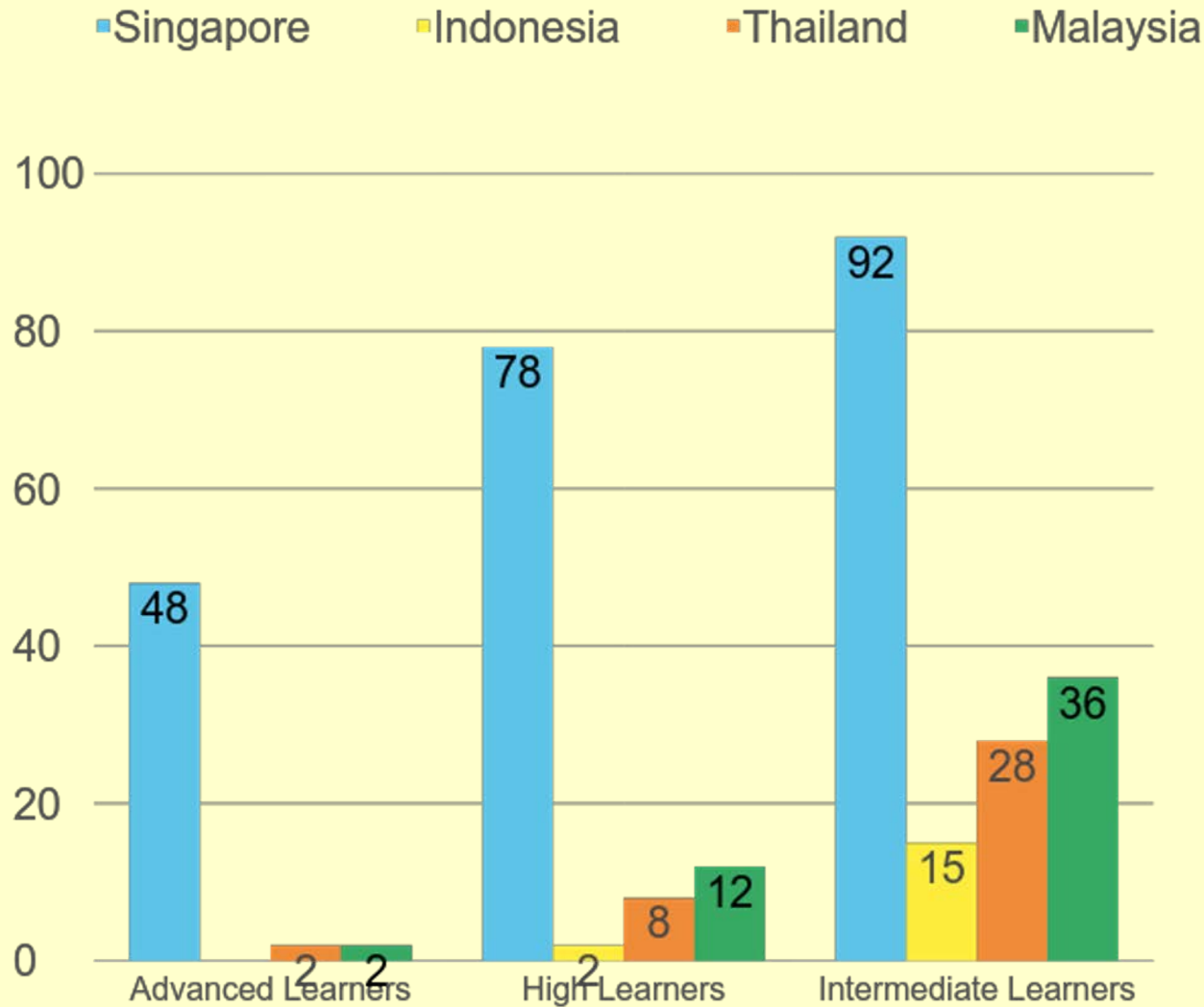
# TIMMS Benchmark 2011



*Grade 8 is the same age as UK Year 9*

*Singapore bucks the trend and maintains its high results in Secondary school.*

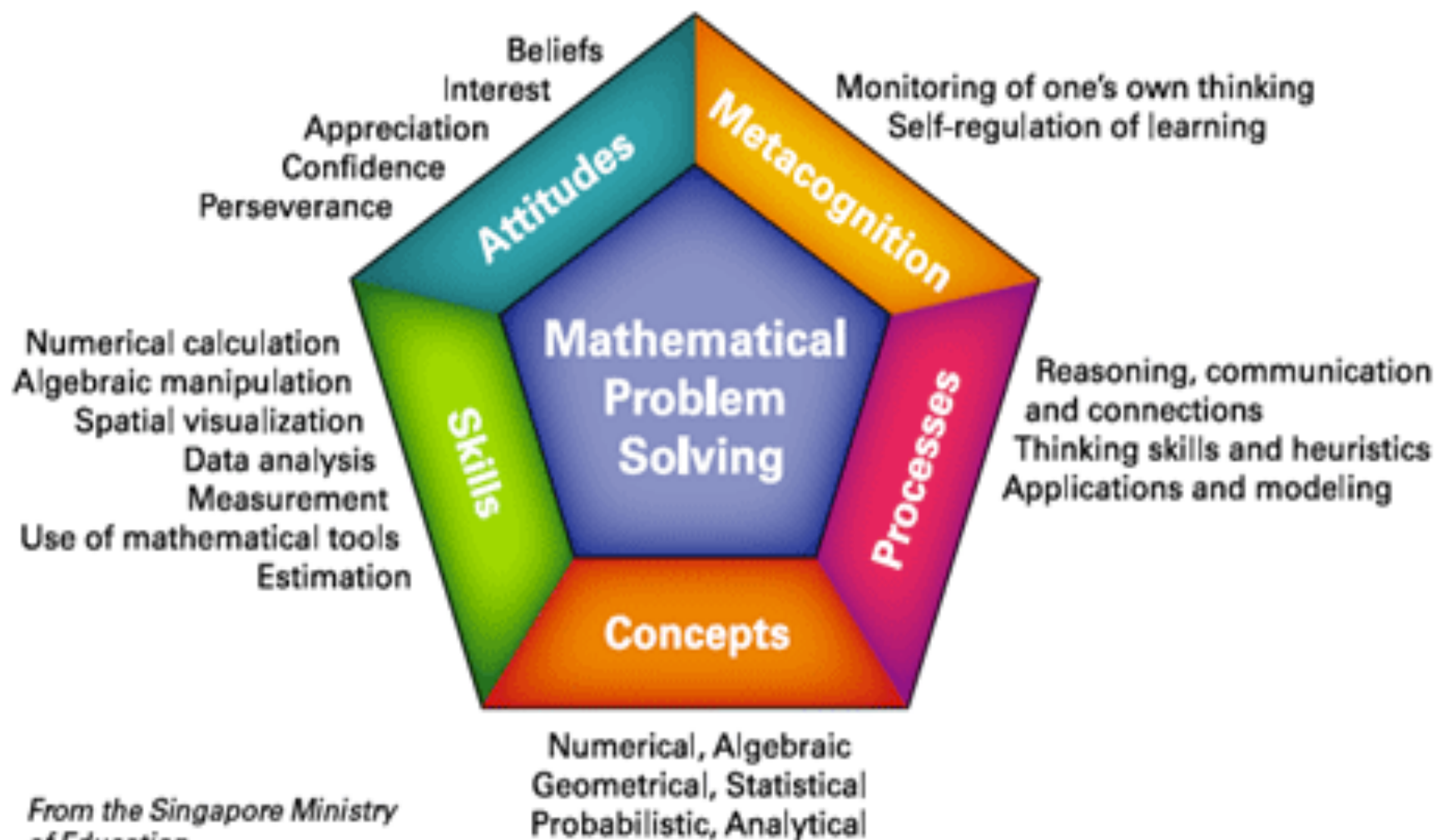
# TIMMS Benchmark 2011



*Grade 8 is the same age as UK Year 9*



*Singapore used to be part of Malaysia and previous to changing how they teach in the 1980s their results were identical to Malaysia*

# Singapore's Mathematics Framework



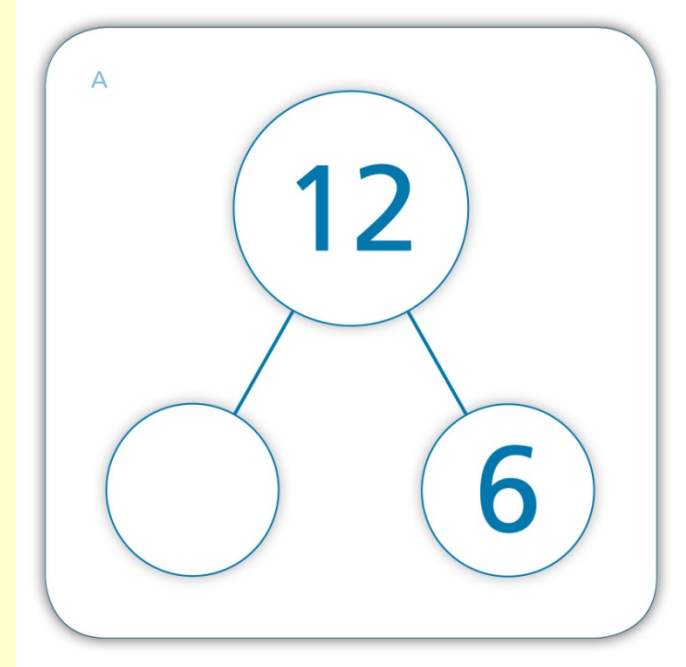
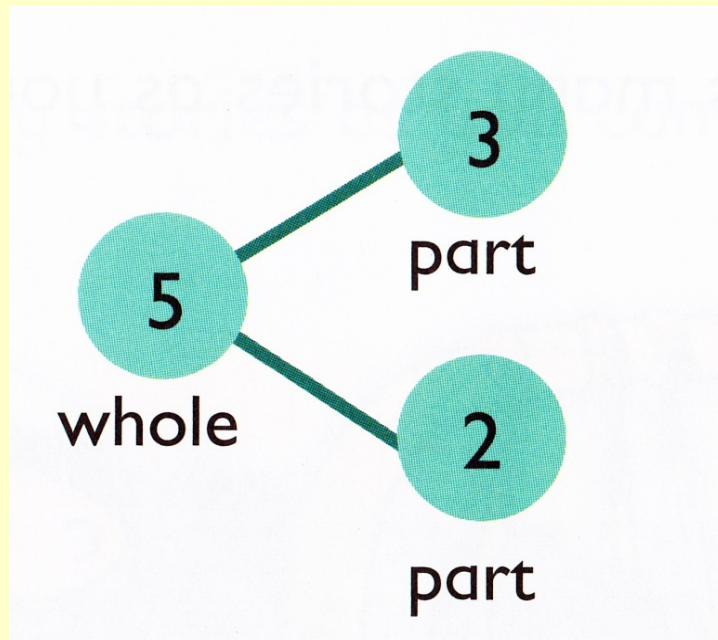
# Big Ideas in Singapore Math

- Number sense
- Making connections and finding patterns
- Communication
- Visualization

Concrete  Pictorial  Abstract

- Variation

# Number Bonds/ Splitting Numbers



# Singapore Maths

- Encourages algebraic thinking even among early learners
- Reduces complexity
- Promotes deeper understanding
- Enhances problem solving
- Helps all students
- Supports struggling learners
- Extends gifted



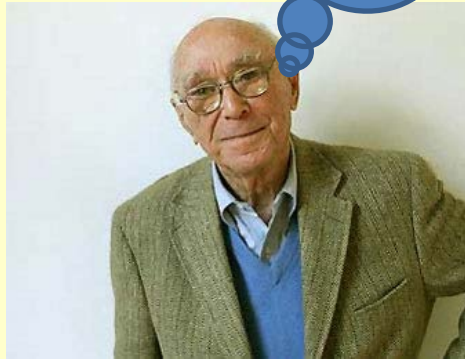
# CPA Model

Jerome Bruner

Enactive

Iconic

Symbolic



Concrete



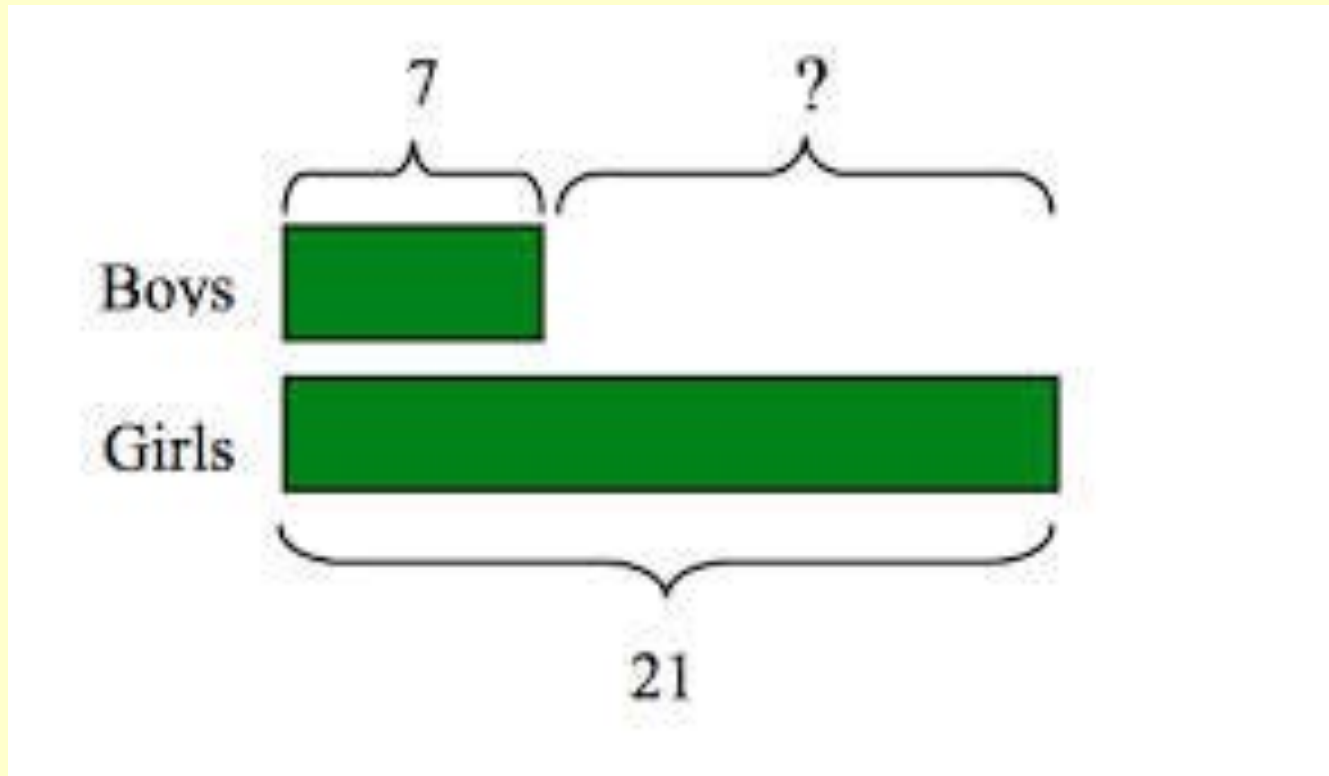
Pictorial



Abstract

$$2 + 1 = 3$$

# Bar Modelling



# Example

There are 9 white flowers. There are 3 times as many red flowers as white flowers. How many red flowers are there?

Two quantities are compared. One is a multiple of the other. We know the smaller quantity. To find the bigger quantity we multiply  $9 \times 3$ .

# Problem Solving

- See
- Plan
- Do
- Look Back

“Bar modeling allows students to engage in algebraic thinking even before they are ready to handle formal algebra.”

-Bar Modeling A Problem Solving Tool, Yeap Ban Har, PhD

# Heuristics

## To give a representation

- Draw a diagram/bar model
- Make a list
- Use equations

## To make a calculated guess

- Guess and check
- Look for patterns
- Make suppositions

## To go through the process

- Act it out
- Work backwards
- Before-after

## To change the problem

- Restate the problem
- Simplify the problem
- Solve part of the problem

# Singapore Maths

## A Typical Lesson

- Mental math activities
- Hands-on lessons with manipulatives
- Partner conversations
- Guided and independent practice



## Differentiation

- 3 different styles of learning: kinesthetic, visual, and auditory
- Depth of knowledge within the same topic

# What makes Singapore Math such a strong curriculum?

- Emphasizes the development of strong number sense, excellent mental-math skills, and a deep understanding of place value.
- A progression from concrete experience using manipulatives, to a pictorial stage, and finally to the abstract level or algorithm.
- Gives students a solid understanding of basic mathematical concepts and relationships before they start working at the abstract level.

Includes a strong emphasis on model drawing, a visual approach to solving word problems that helps students organize information and solve problems in a step-by-step manner.

Because pupils work with concrete apparatus, they are always solving problems rather than just learning algorithms – they become good problem solvers



# Principles

- ‘Five Wise Guys’- Dr Ban Har Yeap
- Piaget- Teach less – learn more
- Bruner- CPA
- Dienes- Informal before formal
- Skemp- Relational Understanding
- Vygotsky- Collaboration

# Using Dot Patterns

Dot patterns bridge the gap between concrete and abstract work

Will help develop a sense of number

Will help develop the concept of conservation of number

Can be linked to familiar patterns eg dice or dominoes

# Key facts and Derived facts


- Consider the UK money system
- 1p, 2p, 5p and 10p
- Why have we chosen these amounts?





# Key Facts

- Key Facts are the 'easy know' facts

The same for every times table

- $2 \times n =$  

$5 \times n =$  

$10 \times n =$  

# Key facts

Multiplication by repeated addition

**The 'Key Facts' are: 1x 2x 5x 10x**

$$3 \times 8 = 8 + 8 + 8$$

**1 x8 and 2x8**

$$6 \times 8 = 8 + 8 + 8 + 8 + 8 + 8$$

**1x8 and 5x8**

$$7 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8$$

**2x8 and 5x8**

$$12 \times 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$$

**2x8 and 10x8**

# Derived facts

If we know that  $5+5=10$  then what else can we derive from this?

$$50+50=100$$

$$5+6=11$$

$$5+4=9$$

# Derived Facts

72 students aged 7 – 13 years... addition

Above average:

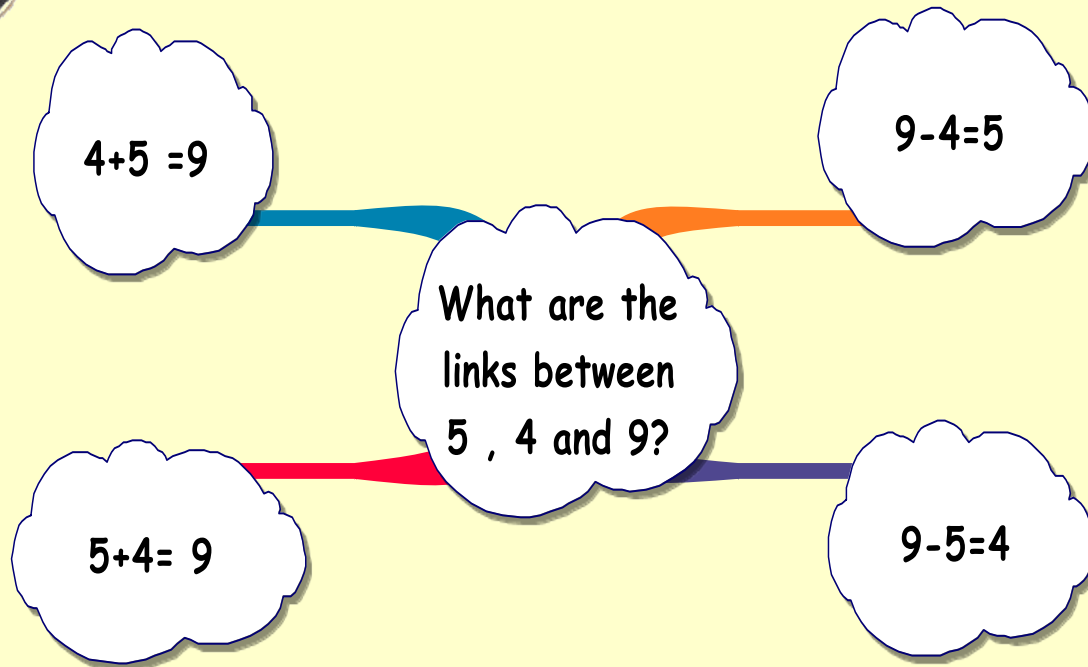
- 9% counted on
- 30% known facts
- 61% derived facts

Below average:

- 72% counted on
- 22% counted all
- 6% known facts
- 0% derived facts

Gray and Tall. 1994. University of Warwick

# Making Links





# Sharma

## Four major principles for teaching

1. Use of Appropriate Concrete Models



2. Levels of Knowing Mathematical Ideas

3. The Three Components of a Mathematical Idea

4 The Questioning Technique

Professor Sharma Berkshire Mathematics



# 1. Use of Appropriate Concrete Models

For early mathematical concepts, it is important that a child experiences mathematics through appropriate and efficient learning models.

Cuisenaire rods, base 10 materials and the Invicta Balance provide appropriate models for these concepts.



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# 2. Levels of Knowing Mathematical Ideas

Intuitive

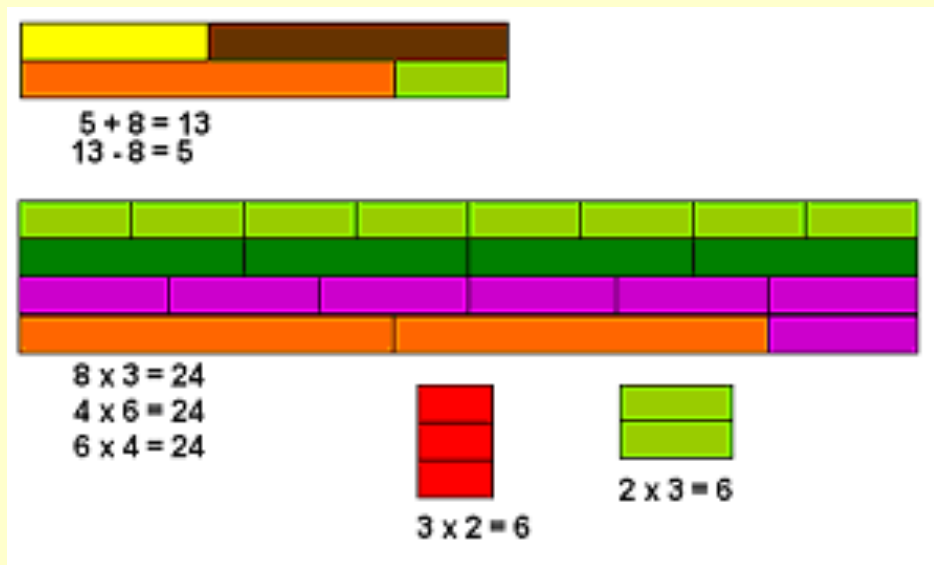
Concrete

Pictorial

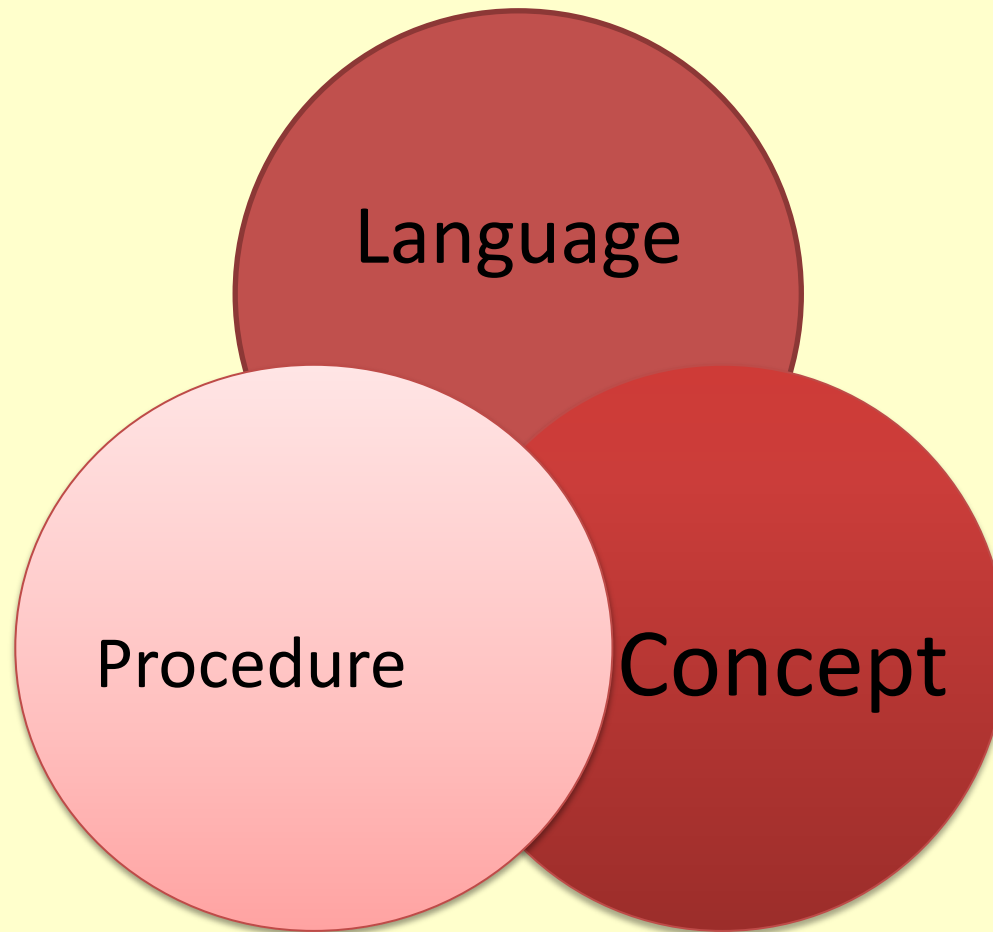
Abstract

Applications

Communication



### 3. The Three Components of a Mathematical Idea



# The importance of mathematical language

- In order to fully understand and grasp mathematical concepts it is vital that children are able to describe what they are doing using mathematical language before they attempt to put it into symbols.
- The more that we can relate the maths that they are doing to the real world the better. This makes it relevant and helps them to make sense of the world

# 4. The Questioning Technique

For the development of concepts, the teaching process must engage the child by asking key questions.

Appropriate questioning is important for the introduction of a concept, for reinforcing it and for helping the child to memorise facts.

# Summary

CPA Approach

Teach less and they will learn more

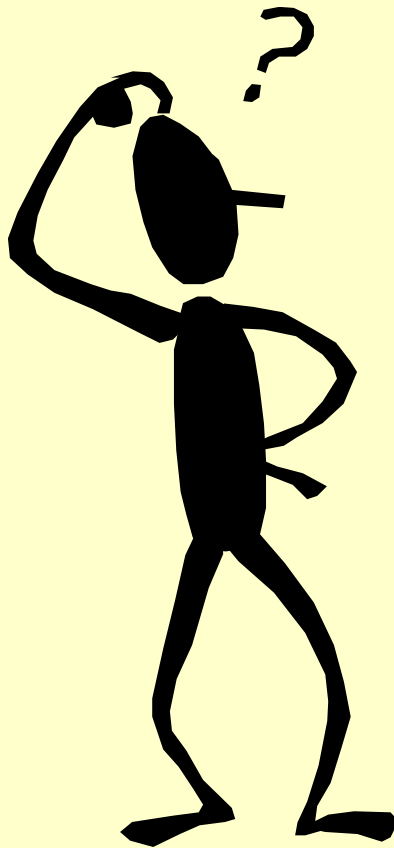
Visualisation

Communication

Number sense

Problem Solving focus

# Any Questions?



Judy Hornigold