What are the important influences for learning number sense, and what are the barriers to achieving this goal?

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SASC. SpLD Assessment Standards Committee

The following 4 principles apply to **Dyscalculia** and **Specific Learning Difficulties** and differentiate them from other mathematics difficulties.

1. Difficulties must be unexpected in relation to age, level of education, level of experience and level of other attainments:

2. Difficulties should be specific and persistent.

3. Difficulties must not be solely caused by other factors such as:

- Inappropriate teaching or gaps in mathematics education
- Social and personal factors which affected attitude/motivation with regard to learning mathematics
- Incomplete mastery of the language of instruction (eg EAL/ESL)
- Mathematics anxiety
- General learning difficulties

4. Difficulties should not arise from another neurological, physical or mental health condition.

What is Dyscalculia? (SASC)

The core feature of Dyscalculia is a domain specific deficit in sense of number. This manifests as difficulties with subitising, symbolic and non-symbolic magnitude comparison, and ordering (cardinality, ordinality).People with Dyscalculia will also have a wide range of other mathematics difficulties: understanding of number is essential to development of arithmetic skills and arithmetic is the first stage of mathematics teaching, so difficulties in this area are likely to have a negative impact upon subsequent mathematics learning. Dyscalculia can co-occur with other SpLDs.

Dyscalculia definition. SASC

Dyscalculia is a specific and persistent difficulty in understanding numbers which can lead to a diverse range of difficulties with mathematics. It will be unexpected in relation to age, level of education and experience and occurs across all ages and abilities.

Maths Learning Difficulties.

Mathematics difficulties are best thought of as a continuum, not a distinct category, and they have many causal factors.

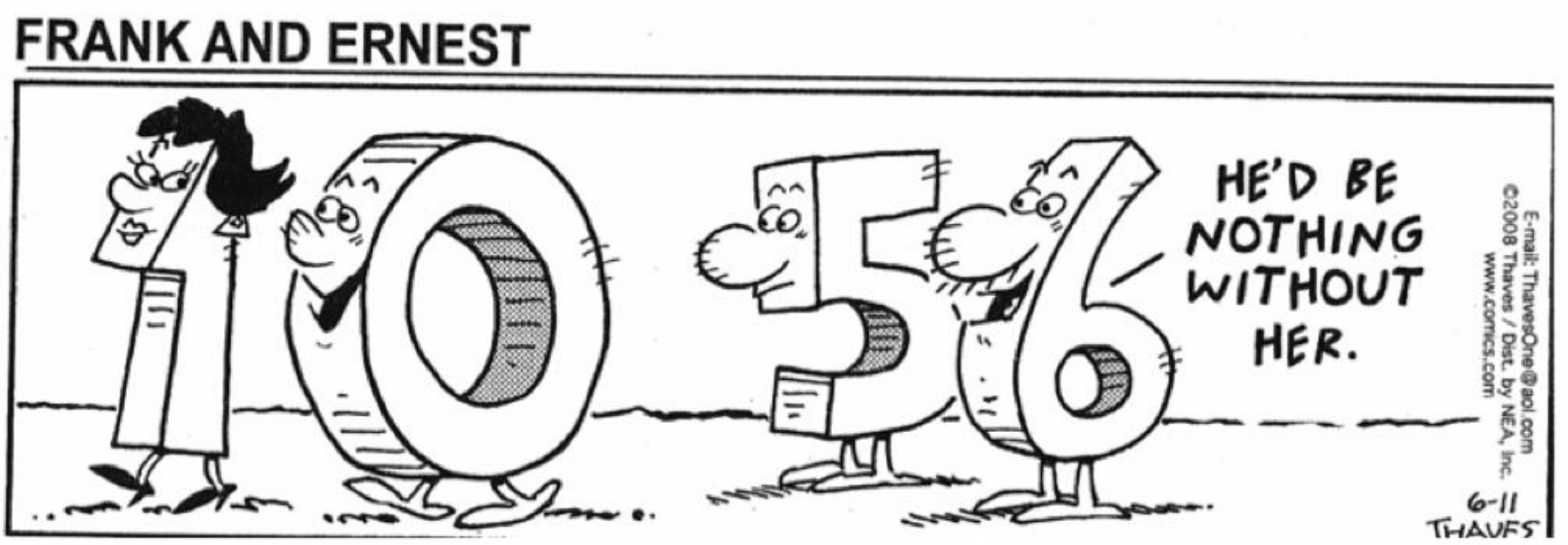
Dyscalculia falls at one end of the spectrum and will be distinguishable from other mathematics issues due to the severity of difficulties with number sense, including subitising, symbolic and non-symbolic magnitude comparison, and ordering.

It can occur singly but often co-occurs with other specific learning difficulties, mathematics anxiety and medical conditions.

- **What should be included in an assessment of difficulties with mathematics?**
- Diagnostic assessment of difficulties in mathematics should form part of a holistic assessment designed to explore the full range of Specific Learning Difficulties and cognitive, medical and environmental factors that may be contributing to difficulties with learning. As such they should include the following:
- A Framework for a thorough and appropriate history-taking which covers mathematics, literacy and wider barriers to learning.
- Tests of verbal and visual/visual-spatial reasoning and cognitive processing (such as memory, phonological processing, processing speeds, visual spatial and visual sequential skills) in order to identify domain general strengths and weaknesses within the cognitive profile.
- Tests of literacy and mathematics skills in order to identify strengths and weaknesses within the attainment profile.
- Informal, qualitative tests of understanding of number that use subitising symbolic and non-symbolic magnitude comparison, ordering and concrete tools to explore concept of number. This could include screeners designed to explore number sense.

- Standardised measures of
- Arithmetic (+,-, x, ÷). Timed and to establish what difference time pressure makes upon performance.
- Mathematics reasoning and problem solving, including word problems to explore whether the difficulties are language related *What aspects of language related are meant here?*
- Qualitative analysis of performance within these tests:
- Analysis of the individual's pattern of errors,
- Observation and questioning about strategies used
- Observation of motivation, determination, perseverance, impulse inhibition, attention, and which tasks were avoided
- Conceptual understanding of any procedures used
- Use of concrete and visual materials – to evaluate to what extent an individual understands basic mathematical concepts, and to explore any differences between what an individual can achieve with and without manipulatives
- Recommendations for interventions and reasonable adjustments should be clearly linked to:
- The individual's difficulties reported in the background information, and evidenced in the assessor's quantitative and/or qualitative analysis of performance in tests.
- The individual's needs within the classroom, course or job. Wherever possible recommendations should be developed collaboratively with relevant mathematics specialists in the individual's school, course or workplace.
- Reasonable adjustments should be appropriately targeted to address the need without potentially giving the individual an unfair advantage. Assessors should bear in mind that adjustments such as use of a calculator or provision of rest breaks can sometimes target the need more effectively than additional time.

Back to Number Sense



When does it begin?

Pre-natal number sense

Maths builds.

Concepts are developed and extended.

Going back. How far?

Linking information to reinforce and retain.

Inconsistencies and confusion.

Pre-empting/addressing misconceptions.

First learning experiences: Then

Buswell and Judd (1925)

Re-visit. Review. Go back.

‘Errors made in the initial stages of a pupil’s contact with the various processes tend to become fixed.

Initial errors persist and repeat themselves even after periods of apparently full *mastery*.’

Inhibition

First learning experiences: Now

'How People Learn' (2000) NRC, USA

Key Finding 1

Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged they may fail to grasp the new concepts and information that they are taught, or they may learn them for the purposes of a test, *but revert to their preconceptions outside the classroom*.

‘Take the little number from the big number.’

$$\begin{array}{r} 33 \\ -\ 16 \\ \hline 23 \end{array}$$

Go back to the basics ….

The beginning …..

In an age-appropriate way

Error analysis

‘Take the little number from the big number.’

$$\begin{array}{r} 33 \\ -\ 16 \\ \hline 23 \end{array}$$

Key Objective. Carry out column addition and subtraction of positive integers less than 10 000 (age 9 years)

10 y 54.5%

13 y 72.8%

15 y 81.8%

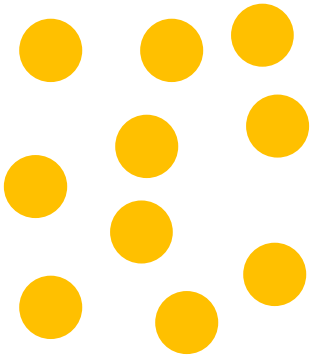
16-19 y 88.3%

What else are you teaching?

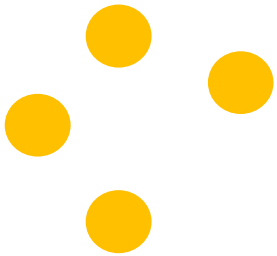
Listening. Classroom ethos.

‘How did you do that?’

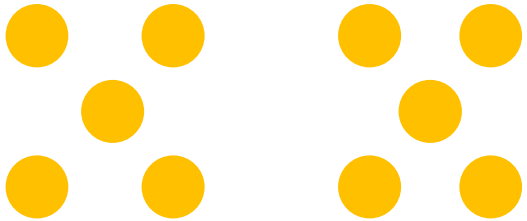
Subitising and counting



counting



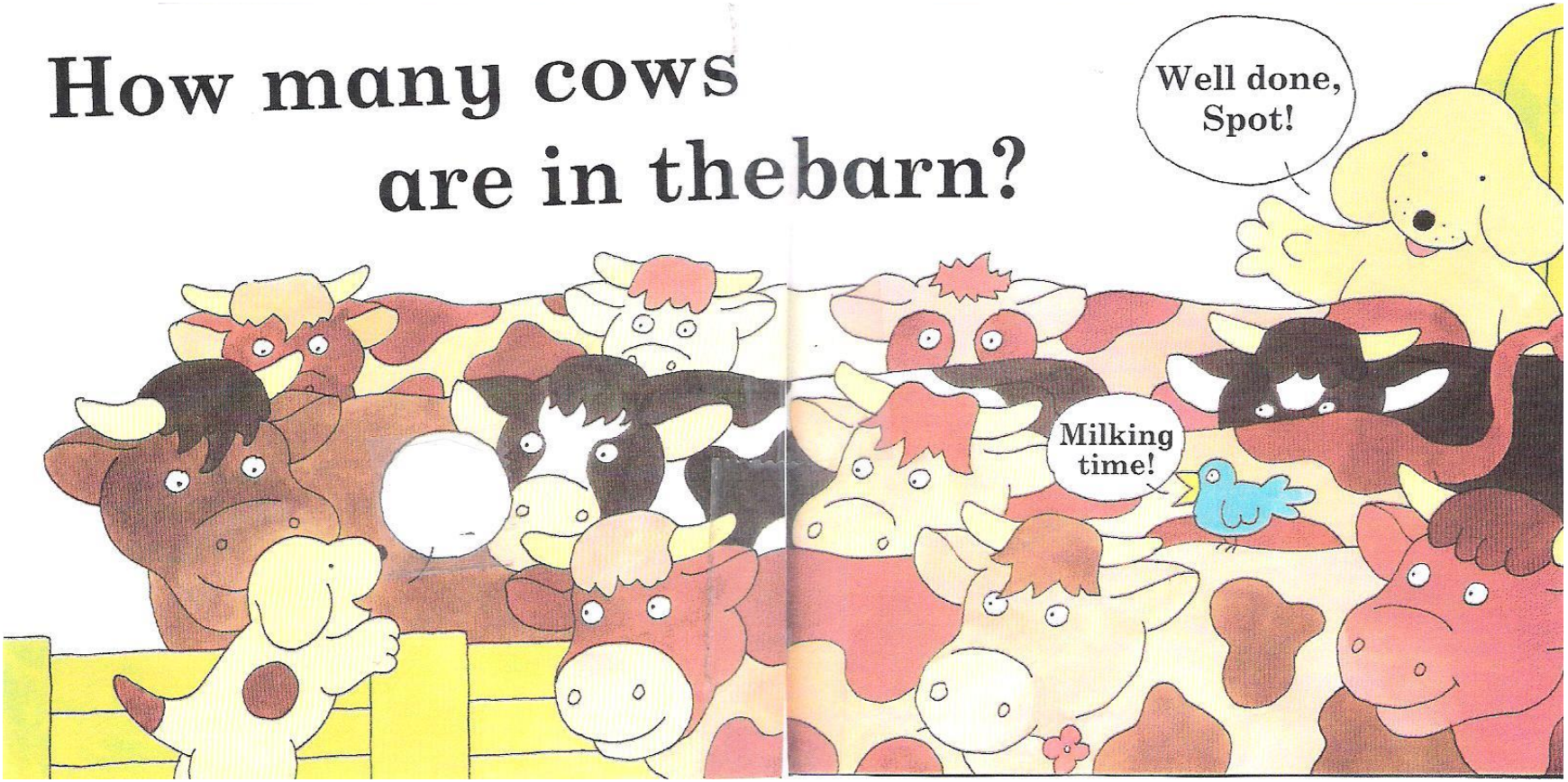
subitising



chunking

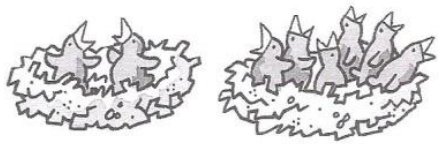
Early teaching of number sense?

clutter

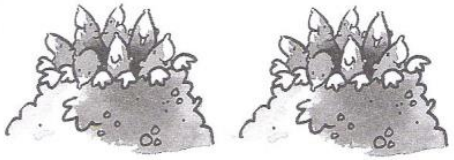


Adding animals

Count and add the animals, then write the number.



2 and 6 → 8



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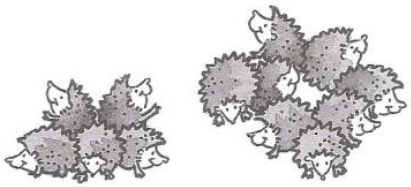

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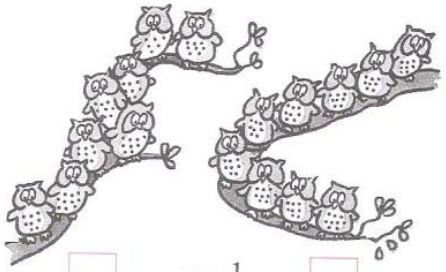

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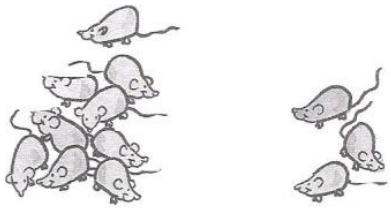

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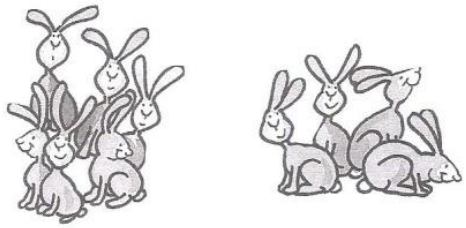


☐ and ☐ → ☐

Is it bigger or smaller?

Maybe

Numerical stroop and **symbols**

7 **5**

8 **2**

4 **9**

The ‘bigger’ number

Vocabulary

Is this a valid ‘test’?

Inhibition

Conflict and Consistency

One digit numbers. Counting

0 1 2 3 4 5 6 7 8 9

10 ?

What else are you teaching?

Bruner's Theory of Development

Enactive	Materials. Base ten blocks + symbols
Iconic	Visual images + symbols
Symbolic	Symbols

Materials/Enactive Learning

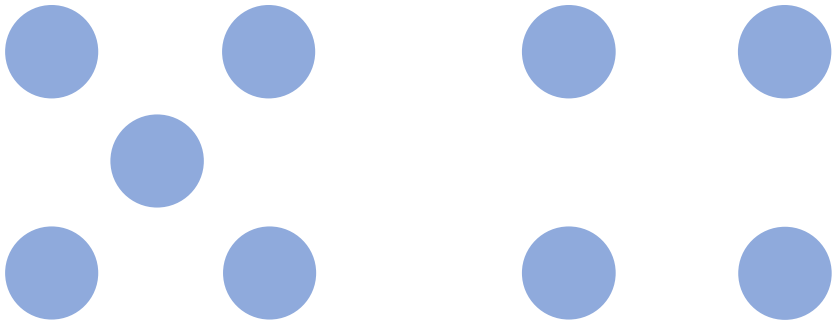
WHY?

LINK

Numicon
'Big Ideas'

Abandoned too soon?

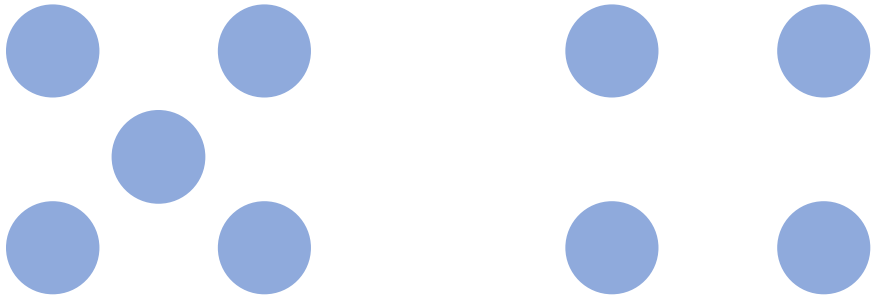
Counting forwards. Adding 1 each time



0 1 2 3 4 5 6 7 8 9

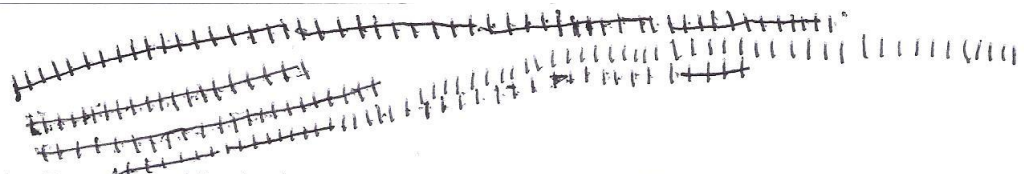
Counting back. Subtracting 1 each time

Reversing. Kruteskii, 1976



9 8 7 6 5 4 3 2 1 **0**

Finger counting…chunking



Maths Test. **15 minutes** (© 2009 S Chinn)

M/F Date 19/05/11 Age 30+ y

1. 2 + 5 = 7
2. 7 + 8 = 21
3. 19 – 4 = 17
4. 5 + 4 + 3 = 12
5. 34 = 4 + 30
6. 400 + 600 = 100000
7. 100 – 58 = 19

‘Skills at using counting procedures to solve addition problems at beginning of 1st grade is NOT predictive of later functional skills.’
Geary, 2013

Preschool Children's Quantitative Knowledge and Long-Term Risk for Functional Innumeracy (Geary 2015)

Children who start school without a solid understanding of number words, numerals, and the quantities they represent are at heightened risk for poor mathematics achievement *throughout schooling* and as a result poor employment-relevant quantitative skills.

Start at the beginning.

Number Sense Berch, 2005

‘No two researchers define number sense in exactly the same way.’

‘Cognitive scientists and math educators define the concept of number sense in very different ways.’

Berch’s literature search resulted in 30 alleged components of number sense.

Selected examples from the 30

An ability to make numerical magnitude comparisons.

An ability to decompose numbers naturally.

Can recognise benchmark numbers and number patterns.

A mental number line on which analogue representations of numerical quantities can be manipulated.

Fractions

Two that are special ….

An ability to make numerical magnitude comparisons.

Possessing knowledge of the effects of operations on numbers.

- ‘Is it bigger or smaller?’

- ‘More’ and ‘Less’ (Donlan, 2014)

More on, ‘An ability to make numerical magnitude comparisons.’

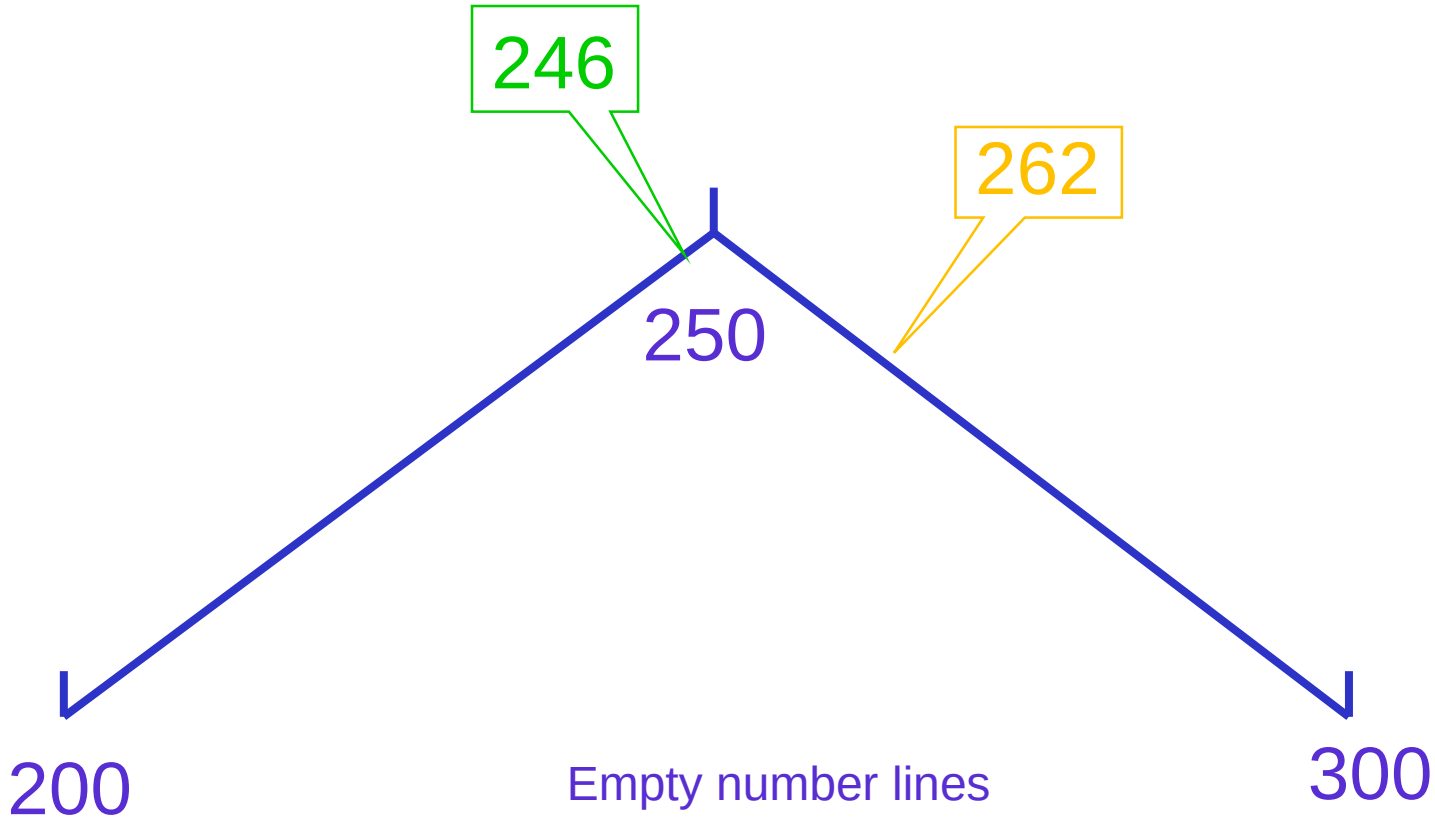
Fractions and the challenge to previous learning

The hidden division sign in fractions.

Renaming, seeing 103 as 90 + 13

Estimation. Link to rounding

Rounding: to the nearest hundred



More on, ‘An ability to decompose numbers naturally.’

Combined with,

Can recognise benchmark numbers and number patterns.

To use: 0 1 2 5 10 20 50 ……

To see: 9 as 10 – 1 98 as 100 – 2

5 as 10 ÷ 2 5% as ½ x 10%

Place value

Foundations. Geary (2015)

At this time, the key foundational competence appears to be *cardinal value; knowing the exact quantities represented by number words and Arabic numerals.*

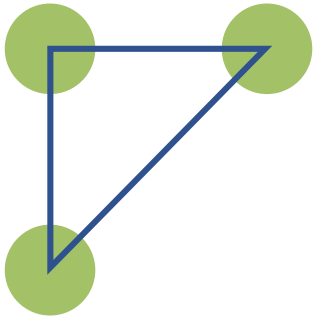
Preschool children's learning of this concept is related to parental background, their intuitive number sense, effortful attentional control and intelligence.

What else are you teaching?
Maths develops.

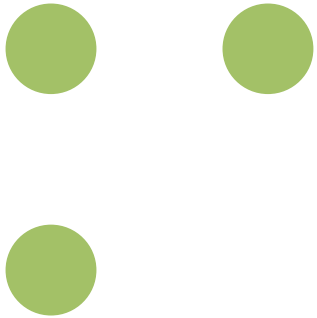
Some key links between numbers

For some students it is good to practise these links using counters

3



A triangle



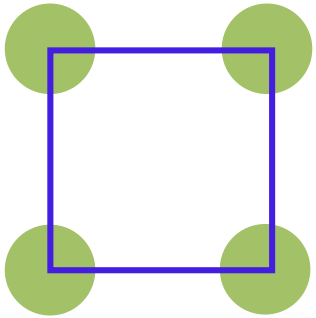
3 = 2 + 1

3 = 1 + 2

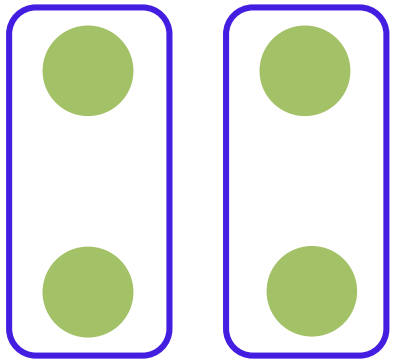
The commutative property

4

Symbols



A square



$4 = 2 + 2$

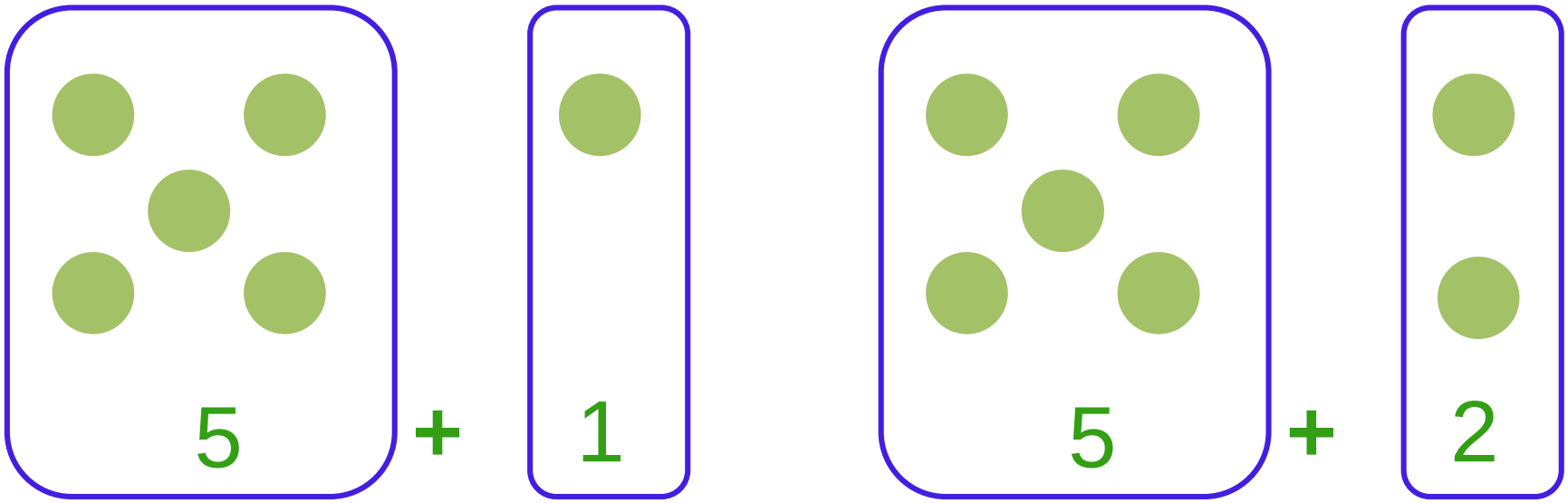
$4 = 2 \times 2$

Linking + and x

$4 = 2^2$

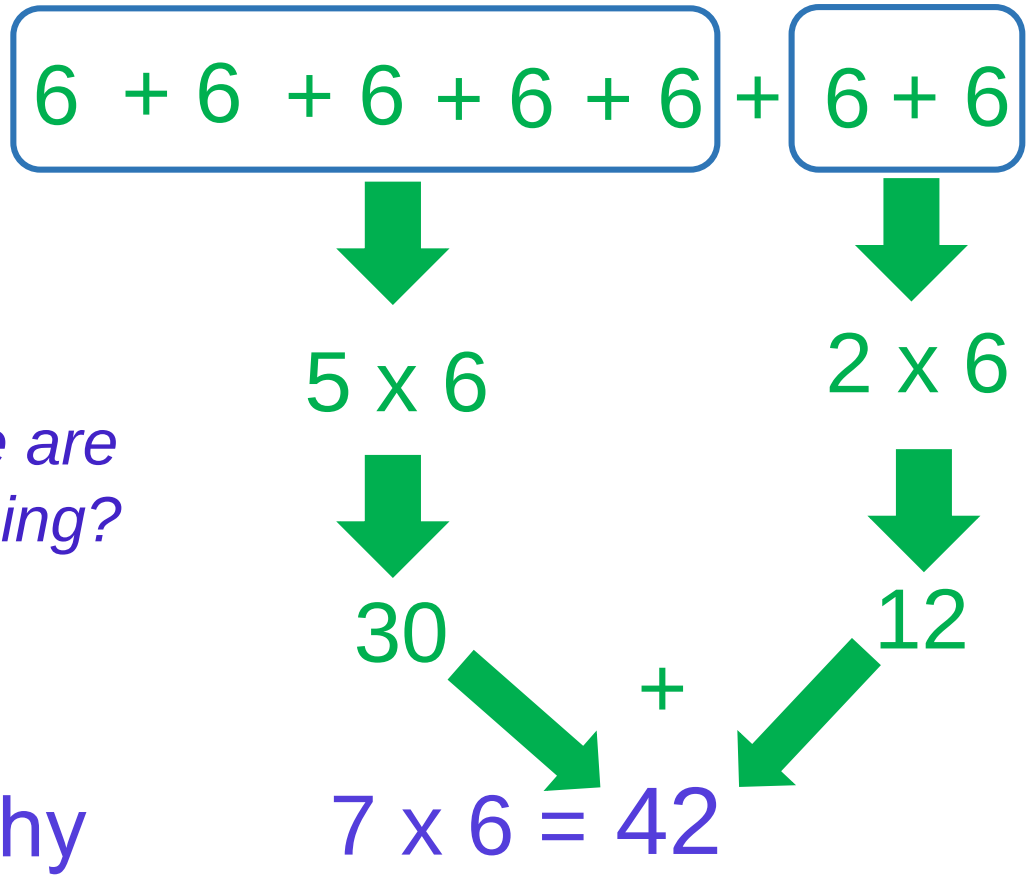
6

7



Chunking.
Using the key numbers to develop numbersense.

Chunking 7 x 6 *partial products*



What else are you teaching?

That's why

A key link between 9 and 10

9 is 1 less than 10

ten

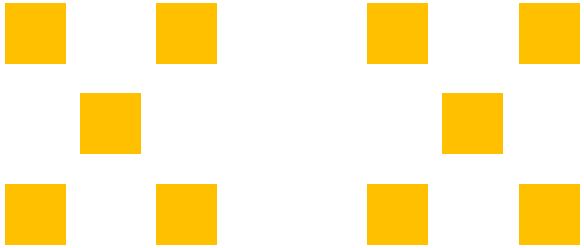
10

99p and 95p

10 - 1

estimation

nine

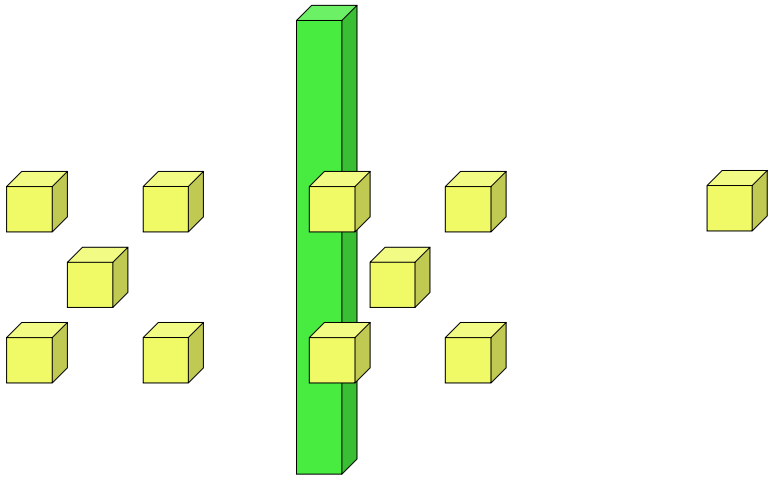


9

Place value.

The digits and where they are placed

eleven



11

one ten

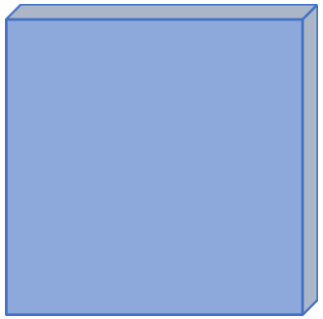
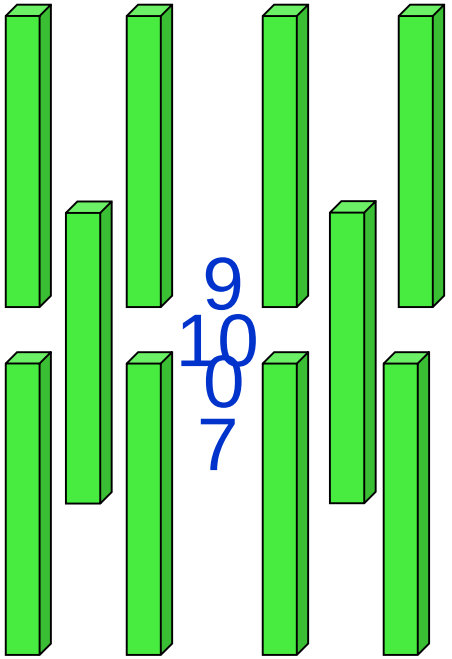
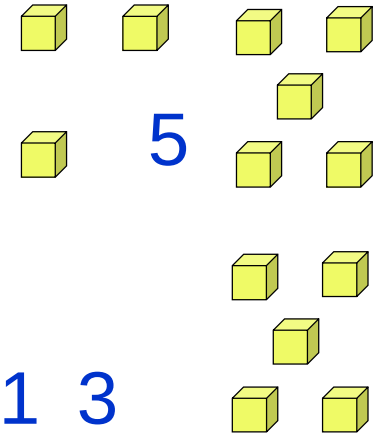
one (one)

Re-naming. Equivalent values. Links.

Renaming (vocabulary) 605 as 500 + 90 + 15

Renaming $\frac{3}{5}$ as $\frac{6}{10}$

Subtracting across zero, an example: 103 - 28

Hundreds 100	Tens 10	Ones 1
		
1	9 10 0 7	1 3

5

$$\begin{array}{r} 1\,\overset{1}{\cancel{0}}\,\overset{1}{\cancel{3}} \\ -\,2\,8 \\ \hline 7\,5 \end{array}$$

Bruner.
Scaffolding

90 + 13 = 103

Subtracting across zero: 103 - 28

H	T	U
1	0	3
	10	3
	9	13
-	2	8
	7	5

1	0	3	
	10	3	
	9	13	
-	2	8	
	7	5	

Subtracting across zero: 103 - 28

10 y 63.6%

13 y 69.6%

15 y 83.6%

16-19 y 87.9%

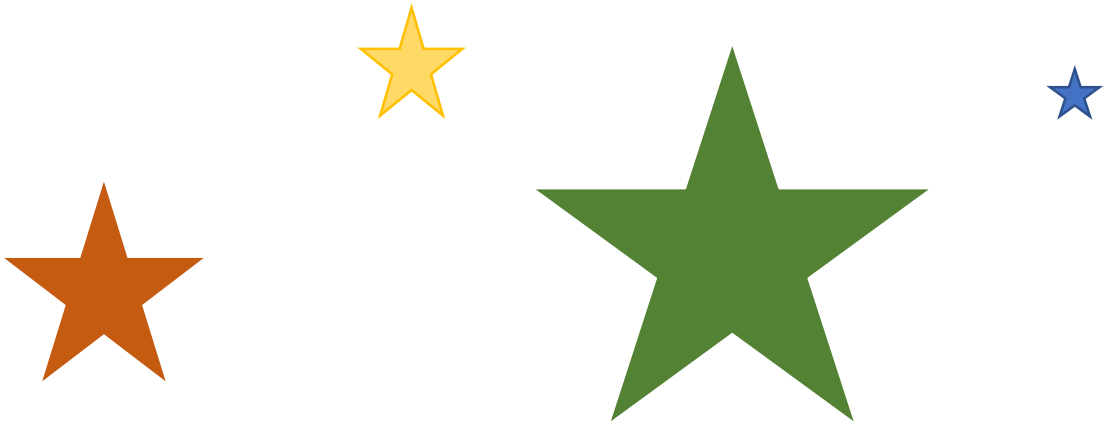
Percentage dyscalculia?

Percentage Maths Learning Difficulties?

Two key skills: Then: Piaget (1941)

Piaget and Szeminska (1941) suggested a relationship between **'seriation'** (or the logical ability to sort objects based on differences while ignoring similarities) and **'classification'** (or the logical ability to sort objects based on similarities, while ignoring differences) and the understanding of number.

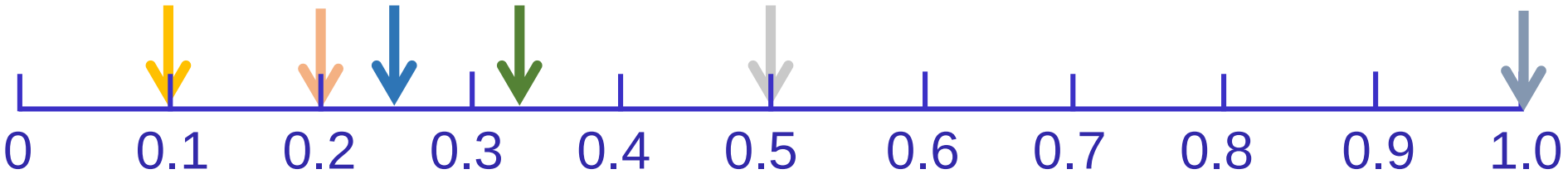
Inhibition

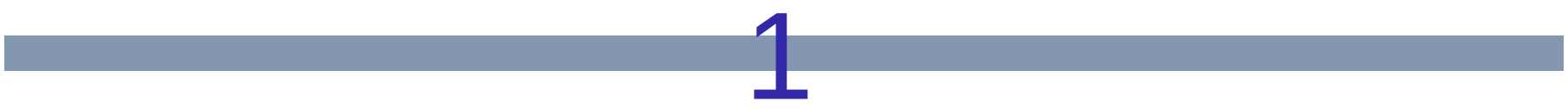


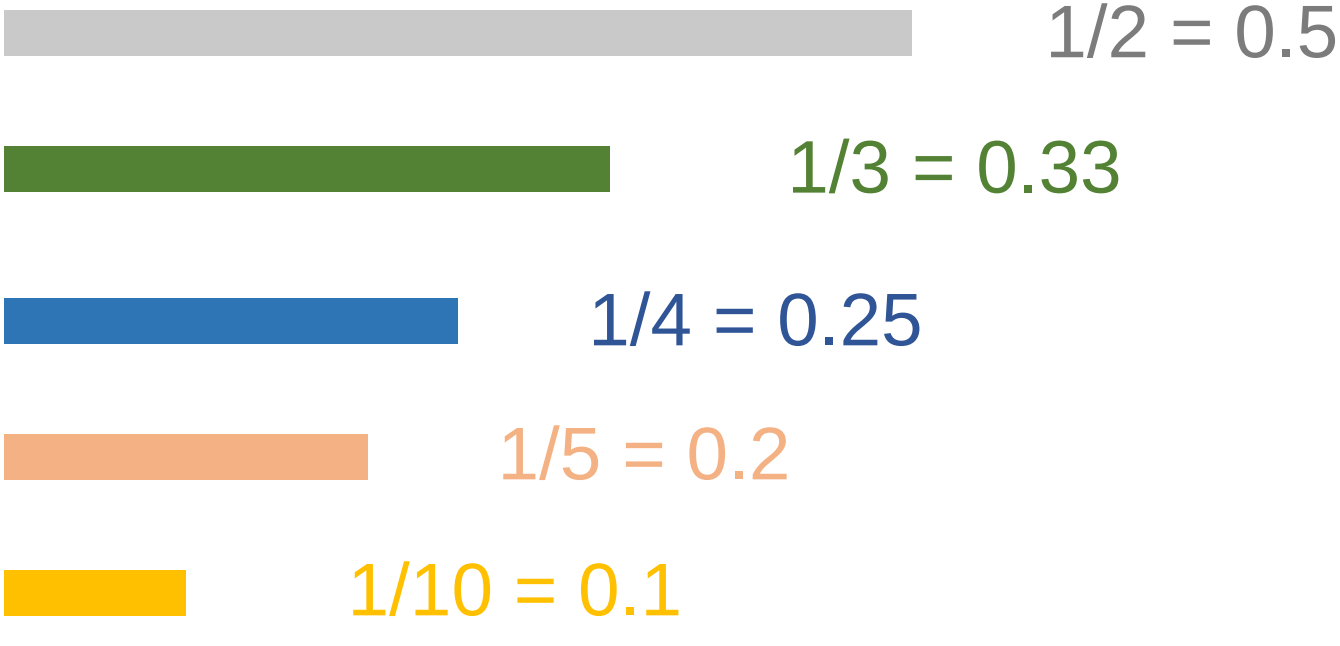
Two key skills: Now: Desoete (2015)

Seriation and classification (as logical thinking skills) and procedural and conceptual counting knowledge are significant **predictive indicators** for mathematical learning disabilities in kindergarten.

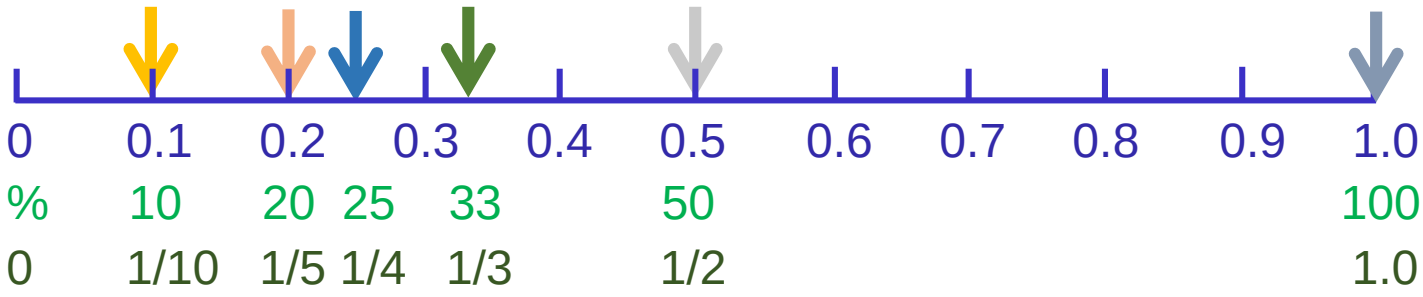
Seriation. Fractions and Decimals







Fractions, Decimals and Percentages



‘How People Learn’

Key Finding 3 (NRC, 2000)

A ‘metacognitive’ approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

‘Can you tell me how you did that?’

Classroom ethos.

Unanxious expectations.

Thank you.