Early Mathematical Skills
Learning and Learning Difficulties: Evidence-Based Assessment and Interventions

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Professor, Special Education, University of Helsinki, Finland
Visiting Professor, University of Oslo, Norway
Visiting Professor, University of Johannesburg, South Africa

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**Introduction of me**

<table>
<thead>
<tr>
<th>PhD Special Education (Beijing, Singapore, Helsinki) 1998-2006</th>
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<tbody>
<tr>
<td>• Development of early mathematical skills and mathematical learning difficulties</td>
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<td>• Early Numeracy Test standardization</td>
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<td>• Mathematics and Thinking skills interventions</td>
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<td>• Children aged 3-8 y.</td>
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<td>• Working memory</td>
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<tr>
<th>Researcher and project manager in Niilo Mäki Institute, University of Jyväskylä 2006-2010</th>
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<tr>
<td>• Development of early mathematical skills and mathematical learning difficulties</td>
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<td>• Computer based interventions for children with mathematical learning difficulties</td>
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<td>• Assessment tools (math learning)</td>
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<td>• Research based web-base for educators, psychologists and parents <a href="http://www.lukimat.fi">www.lukimat.fi</a> (funded by the National Ministry of Education and Culture)</td>
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<th>University Lecturer and Professor in University of Helsinki 2010-present</th>
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<tr>
<td>• Interventions for children with mathematical learning difficulties</td>
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<td>• Johannesburg, South Africa</td>
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<td>• Oslo, Norway</td>
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<td>• Mathematical skills development and related motivational and well-being factors</td>
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<td>• Age group 9-16 y.: screener, test and interventions</td>
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Our research group in Helsinki & Oslo & Johannesburg
Why it is so hard for some children to learn math?

Developmental dynamics in mathematical learning

(Special) educational interventions for children to support the learning

Assessment tools – how to detect the development and learning difficulties

Interest, motivation, well-being, executive functions

Age of students 5-18 years

To make learning easier for those who struggle -> EDUCATORS ARE IMPORTANT
**Children with special educational needs in mathematics**  
(Aunio, 2014)

- **Mathematical learning disability**
- **Dyscalculia**
- **Mathematics disorder**  
  ICD-10: 5-7%

- **Mathematical learning difficulties**
- **Low achievement in mathematics**
- **Low performance in mathematics**  
  15-20%

- **Average performance**

Geary (2013): 2 consecutive years, MLD under 10th percentile; LA 11th-25th percentile

Intensity, specificity and time needed grows

Berch & Mazzocco 2007,  
Geary 2013; Price & Ansari 2013
## Topic of today

<table>
<thead>
<tr>
<th>Topic</th>
<th>Other groups</th>
<th>Aunio et al. group</th>
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<tbody>
<tr>
<td>Models of development</td>
<td>Fritz et al. 2013</td>
<td>Core Factor Model 5-8 years</td>
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<td>Krajewski &amp; Schneider 2009</td>
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<td>Clements &amp; Sarama 2009</td>
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<td>Steffe 1992</td>
<td>Core Factor Model 9-12 y</td>
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<td>Wright et al. 2006</td>
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<td>Assessment</td>
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<td>Early Numeracy</td>
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<td>LukiMat-screeners</td>
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<td>CodyTest</td>
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<td>Interventions</td>
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<td>ThinkMath-interventions</td>
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Research into

DEVELOPMENT OF MATHEMATICAL SKILLS
Attempts to model mathematical development in age group 5-8 years

- Fritz et al. (2013) constructed a hierarchical framework of competences grounded on earlier theoretical and empirical findings (average performing).

- Krajewski and Schneider (2009) presented a theoretical idea about the transition from a procedural to an increasingly conceptual understanding of number words via three levels (average performing).

- Sarama and Clements (2009) is based on theory and research on early childhood learning and teaching, with a focus on children’s learning trajectories (average performing).

- Steffe’s (1992) framework has a constructivist view on learning and was based on individual children’s mathematical development (average performing).

- The Steffe’s framework was a starting point for Wright et al., (2006), whose model has been used for providing individual support for low-performing children.
Our aim was

- What are the most important numerical skills for children aged 5-8 years old? (so that no mathematical learning difficulties will appear later)

- An evidence-based model for educators to help to work with young children who are at risk for learning difficulties
We solved the problem:

- Longitudinal studies 2000-2015
- Assessment batteries:
  - should be published with norms
  - used by educators
  - Focus on multiple mathematical skills relevant in this age group of children
  - Not curriculum depended
  1) Utrecht Test of Early Numeracy (Van Luit et al)
  2) Number Knowledge Test (Griffin 2003)
  3) Early Numeracy Test (Wright et al. 2006)
  4) Test of Early Mathematics Ability (Ginsburg & Baroody 2003)
European Early Childhood Education Research Journal
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/recr20

Core numerical skills for learning mathematics in children aged five to eight years - a working model for educators

Pirjo Aunio\textsuperscript{a} & Pekka Räsänen\textsuperscript{b}
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\textsuperscript{b} Niilo Mäki Institute, University of Jyväskylä, Jyväskylä, Finland
Published online: 12 Jan 2015.
Understanding mathematical relations

- early mathematical-logical principles
- basic arithmetic principles
- operational symbols in mathematics
- place-value and base-10 system

Counting skills

- knowledge of number symbols
- number word sequence
- enumeration

Basic skills in arithmetic

- arithmetic facts
- addition and subtraction

Symbolic and non-symbolic number sense
Nonverbal number sense (1)

- Operationalized: Subitizing, pattern recognition, magnitude comparison (some sort of sense making of numerical magnitudes)

- Number sense (two ways to use):
  - Educational scientists: A mélange of skills learnt before formal education (e.g., counting, number patterns, magnitude comparisons, estimating and number transformations) (Jordan, et al. 2007)
  - Neuroscientists: restricted to concept to cover only those numerical abilities that are common to animals and humans from infants to adults (e.g., approximate representation of magnitude) (Dehaene, 1997/2011; Lipton & Spelke 2003)

-> number sense lays a foundation for learning formal mathematical concepts
Nonverbal number sense (2)

- Weakness in number sense
  - Crucial role when difficulties in mathematical learning (Mazzocco, Feigenson & Halberda 2011a; Price & Ansari 2013)
    - Prediction power (approximate number system) to later learning of mathematical skills (Libertus, Feigenson & Halberda 2011; Mazzocco, Feigenson & Halberda, 2011b)
    - Mental representation of the magnitude of symbolic numbers or representation of non-symbolic magnitudes (De Smedt, Nöel, Gilmore & Ansari, 2013)
Understanding mathematical relations (1)

- Quantitative and non-quantitative relationships

  I Mathematic-logic principles:
  - Seriate (Bryant, 1996)
  - Numerically compare two sets (Sophian 1998)
  - Classify (Smith, 2002)
  - One-to-one correspondence (Alibali & DiRusso, 1999)
II Basic-arithmetic principles, often the understanding of part-whole relations in addition or subtraction tasks (Canobi, Reeve & Pattison 2002; Wilkins, Baroody & Tiilikainen 2001):

- **Additive composition principle**: understands that the larger sets are made up of smaller sets
- **Commutativity understanding**: knows that entities added can be added in a different order and still have the same answer, \( a+b=b+a \)
- **Associativity understanding** (i.e., decomposition, transforming strategy): knows that even if the sets are decomposed, and recombined in different order, they have the same answer, \( (a+b)+c=a+(b+c) \)
- **Inversion**: understanding the relationship between two different operations such as that addition and subtraction rule each other out, knowing the results in addition can be used to solve the subtraction problem (Robinson, Niowski & Gray, 2006)
III Understanding operational symbols in mathematics:

- less than (<)
- more than (>)
- equal to (=)
- not equal to (≠)
Understanding mathematical relations (4)

IV Understanding place-value and base-10-system

- Understanding of meaning of e.g., ones, tens and hundreds in the base-10 system
  - The number symbols have different value depending on the place they occupy in the series of numbers
  - The ability to match verbal labels to Hindu-Arabic numerals (i.e., Number sequence skills) appears generally to precede single- and double digit comprehension, but the findings suggest that in the acquisition of place-value knowledge, there is also non-verbal numerical knowledge involved (Donlan & Gourlay 1999)

Counting skills (1)

- I knowledge of number words and symbols
  - Learning the number words
    - If culture does not have symbols representing exact quantities, without number words and tradition of counting, even adults are not able to do simple imitation or copying (one-to-one correspondence) tasks accurately (Gordon, 2005; Pica et al. 2005)
    - Symbol-verbal and verbal-symbol transitions = word identification and recognition of numbers (e.g. choose number in VanDerHeyden et al. 2006)
Counting skills (2)

Il Number words sequence skills (NWS, synonyms often acoustic counting, reciting number words, counting) (VanDerHeyden et al. 2006; Clarke and Shinn 2004)

- NWS-skills include:
  - ability to say number words forward and backward
  - by skipping for instance by twos, fives and tens
  - Saying number words forward and backwards from a given number
Counting skills (3)

III Enumeration (often counting the numerosity of a set, counting, cardinal meaning of number, counting objects)

- Child uses her/his number word sequence skills to enumerate (Aunio & Niemivirta 2010; Jordan et al. 2006)
  - To say NWS in correct order
  - To be able to create the one-to-one correspondence between number words and items to be counted + between number words and a marking act + between the marking act and items to be counted
  - To understand that the just named number word states the numerosity of the set
  - To know that objects of any kind can be collected together and counted
  - To know that objects of any kind can be collected together and counted and that items within a given set can be tagged in any sequence.
# Development of counting skills

<table>
<thead>
<tr>
<th>App. age</th>
<th>Phase</th>
<th>Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 y.</td>
<td>Primary understanding of amounts</td>
<td>different number words refers to a different number of objects, only very basic discrimination of amounts</td>
</tr>
<tr>
<td>3 y.</td>
<td>Acoustic counting</td>
<td>say number words but not in correct order</td>
</tr>
<tr>
<td>4 y.</td>
<td>Asynchronous counting</td>
<td>Say number words in correct order and point to objects, pointing is not coherent</td>
</tr>
<tr>
<td>4,5 y.</td>
<td>Synchronic counting</td>
<td>Can recite number words and mark the counted objects correctly, by pointing or moving</td>
</tr>
<tr>
<td>5 y.</td>
<td>Resultative counting</td>
<td>Can say number words correctly starting with one, understand that countable objects should be makers one, last said number word indicates the number of objects</td>
</tr>
<tr>
<td>5,5 y.</td>
<td>Shortened counting</td>
<td>Recognize the figure five, for instance and can continue counting upwards from that</td>
</tr>
</tbody>
</table>
Basic Skills in Arithmetic

At this age group mastering addition and subtraction with number symbols have been covered.

Three types of tasks:

- The question is presented verbally and manipulative or pictures are used to ease the memory load so that children can point to the correct answer from couple of possibilities.
- Some or all of the items are unscreened (i.e., covered).
- Only verbal prompts are used; the question is asked or shown in written form.

• The difficulty level depends also on the magnitude of numbers used in the tasks.
Aunio, P., Laine, A. & Räsänen, P. (manuscript in preparation) Development of mathematical skills in 9-12 years old children
The aim of the study & method

What are the core skills in mathematical development in grades 3-6 (9-12 years)?

Phase one:
- Review of existing assessment batteries for children’s mathematical skills in grades 3-6 (9-12 years)
- Categorizing the skills measured

Phase two:
- Review of longitudinal studies measuring children’s math skills in grades 3-6 (9-12 y.)
Phase one – “data”

**Finnish**
3. Häyrinen, Serenius-Serve & Korkman (2013) Lukilasse-2. 7-12 years old

**International**
3. Von Aster, Weinhold Zulauf & Horn (2006) ZAREKI-R (n 7-16 y.) leerjaar 3-> 2 klas voortgezet onderwijs
Potential core skills in mathematics in age group 9-12 years

Rational numbers

Natural numbers

Counting skills

Number Sense

Arithmetic skills

Fluency

Addition skills

(Subtraction skills)

Word problems

Multiplication skills

(Division skills)
Evidence-based
ASSESSMENT BATTERIES IN MATHEMATICS
Recent development with early numeracy assessment

- Long & brief measurement
- Curriculum Based Measurements (CBM)
- Computer assisted assessment
### Long and Brief measurement

- To identify young children with mathematics difficulties
- Test & screeners
- Jordan et al. 2006 (long), 2007 (brief)

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting</td>
<td>One-to-one correspondence, Knowing stable order and cardinality principles, Knowing the count sequence</td>
</tr>
<tr>
<td>Number knowledge</td>
<td>Discriminating and coordinating quantities</td>
</tr>
<tr>
<td>Number transformations</td>
<td>Transforming sets through addition and subtraction, Calculating in verbal and nonverbal context, Calculating with and without referents (physical or verbal)</td>
</tr>
<tr>
<td>Estimation</td>
<td>Approximating or estimating effect sizes, Using reference points</td>
</tr>
<tr>
<td>Number patterns</td>
<td>Copying number patterns, Extending number patterns, Discerning numerical relations</td>
</tr>
</tbody>
</table>
Long and Brief measurement
- Screening + more information

<table>
<thead>
<tr>
<th>Author</th>
<th>Name of the test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purpura et al. (2015)</td>
<td>Preschool Early Numeracy Skills Screener-Brief</td>
</tr>
<tr>
<td>Siebert &amp; Brendefur 2018 (Brenderfur et al. 2015; Brendefur, Strother &amp; Thiede, 2012)</td>
<td>The Primary Math Assessment (PMA) (PMA-Screener; PMA-Diagnostic (number sequencing, operations (number facts) contextual problems, relational thinking, measurement and spatial reasoning))</td>
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</table>
## Curriculum Based measurements

To inform instruction and interventions

<table>
<thead>
<tr>
<th>Authors</th>
<th>Name of test battery</th>
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<tbody>
<tr>
<td>Starkey, Klein &amp; Wakeley 2004</td>
<td>The Child Math Assessment (CMA)</td>
</tr>
<tr>
<td>Polignano &amp; Hojnoski 2012</td>
<td>CBM tasks for preschoolers (Cardinality, Pattern Completion, Shape Naming Fluency, Shape Selection Fluency, Shape Composition)</td>
</tr>
<tr>
<td>VanDerHeyden, Broussard &amp; Cooley 2006</td>
<td>CBM preschool (Count objects, Choose number, Discrimination, Number naming, Free count)</td>
</tr>
<tr>
<td>Laracy, Hojnoski &amp; Dever 2016</td>
<td>IGDIs-EN (Individual Growth and Development Indicators – Early Numeracy (Quantity comparison, Oral counting, One-to-one correspondence)</td>
</tr>
<tr>
<td>Lee 2016</td>
<td>Birthday Party (Number and operation, Shape, Space and Pattern) (short + long)</td>
</tr>
<tr>
<td>Conoyer, Foegen &amp; Lembke 2016</td>
<td>Early Numeracy Curriculum Based Measurement (CBM) (Quantity Discrimination, Measurement)</td>
</tr>
<tr>
<td>Clarke &amp; Shinn 2004</td>
<td>CBM measurement (Oral counting, Number Identification, Quantity Discrimination, Missing number)</td>
</tr>
<tr>
<td>Lee &amp; Lembke 2016</td>
<td>CBM (quick retrieval, written computation, number sense)</td>
</tr>
<tr>
<td>Lembke, Lee, Park &amp; Hampton 2016</td>
<td>CBM (Counting, Number Identification, Missing Number, Quantity Discrimination, Number Facts)</td>
</tr>
</tbody>
</table>
Curriculum Based measurements

Progress monitoring

Lei, Wu, DiPerna & Morgan 2009:

- EARLI (Early Arithmetic, Reading and Learning Indicators) Numeracy Measures (Numbers and Shapes, Measurement)
<table>
<thead>
<tr>
<th>Test/Screener</th>
<th>Skills</th>
<th>Age group</th>
<th>Authors/source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lukukäsitetesti (Early Numeracy Test) (Fin)</td>
<td>Math relational skills &amp; counting skills</td>
<td>4-7 years</td>
<td>Van Luit, Van de Rijt &amp; Aunio, 2006, Hogrefe Psykologien kustannus</td>
</tr>
<tr>
<td>LukiMat-scales (Fin/Swe)</td>
<td>Number sense, math relational skills, counting skills, basic arithmetic skills</td>
<td>K-2 graders</td>
<td><a href="http://www.lukimat.fi">www.lukimat.fi</a> oppimisen arviointi</td>
</tr>
<tr>
<td>BANUCA (Fin)</td>
<td>number knowledge, arithmetic skills</td>
<td>1-3 graders</td>
<td>Räsänen (2005), Niilo Mäki Insitituutti</td>
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<tr>
<td>RMAT (Fin/Swe)</td>
<td>Arithmetics skills (Fluency)</td>
<td>3-6 graders</td>
<td>Räsänen (2004), Niilo Mäki Insitituutti</td>
</tr>
<tr>
<td>MATTE (Fin)</td>
<td>Word problem solving</td>
<td>4-5 graders</td>
<td>Kajamies, Vauras, Kinnunen &amp; Iiskala (2003) Turun yliopisto, Oppimistutkimuksen keskus</td>
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<tr>
<td>KTLT (Fin/Swe)</td>
<td>Arithmetic skills (application)</td>
<td>7-9 graders</td>
<td>Räsänen &amp; Leino, (2004) Niilo Mäki Insitituutti</td>
</tr>
</tbody>
</table>
The Early Numeracy Test in Finnish: Children's norms

PIRJO AUNIO¹, JARKKO HAUTAMÄKI¹, PEKKA HEISKARI² and JOHANNES E. H. VAN LUIT³

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DOI: 10.1111/j.1467-9450.2006.00538.x

Scandinavian Journal of Psychology
Early Numeracy Test (4 – 7 y. 6 m.)

LukiMat- Math assessment

- For educators (kindergarten, first and second grade)
- To be used with group of children (not individually)
- To find those children who are potentially at risk to have problems with mathematics learning (screening)
- Validated and reliable tools (evidence based)
- Easy to use and analyze
- Free of charge
- Finnish and Swedish
LukiMat- kindergarten norms


<table>
<thead>
<tr>
<th>Persentiili</th>
<th>Syksy (n = 563)</th>
<th>Talvi (n = 504)</th>
<th>Kevät (n = 486)</th>
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</table>
Evidence-based
INTERVENTION PROGRAMS IN
MATHEMATICS
Recent Development with Early Numeracy Interventions (Aunio, in press)

- High demand of randomized control trials (RCT), large scale interventions and replication studies (e.g. Gersten, Rolfhus, Clarke, Decker, Wilkins & Dimino, 2015)

- Meta-analysis and systematic reviews

- Several meta-analysis including students with learning difficulties in mathematics (Chodura, Kuhn & Holling, 2015; Codding, Burns & Lukito, 2011; Gersten et al. 2009; Jitendra et al. 2018; Kroesbergen & van Luit, 2003; Maccini, Mulcahy & Wilson 2007; Zhang & Xin, 2012) but only few have concentrated on young children (Dennis et al. 2016; Mononen, Aunio, Koponen & Aro 2014)

Recent Development with Early Numeracy Interventions (Aunio, in press) (2)

Mononen et al (2014):

- small to average effect sizes in improvement of the early numeracy skills of children aged 4-7 years

- instrucational features:
  * explicit instruction,
  * computer-assisted instruction (CAI)
  * game playing
  * use of concrete-representational-abstract levels in representations of mathematical concepts
Recent Development with Early Numeracy Interventions (Aunio, in press) (3)

Dennis et al. (2016)

- kindergarten level interventions weaker effects than those at elementary level
- students with low performance in mathematics (at or below 35th percentile) – stronger effects than those above 35th percentile
- more effective when conducted by the researcher or research assistant, than by teachers
- instructional features: peer-assisted learning, explicit teacher led instruction (i.e. sequencing tasks from easy to difficult, task analysis), use of technology
- replicates the findings, at least partly in other meta-analysis (e.g. Baker, Gersten & Lee 2002; Swanson et al. 1999)
Recent Development with Early Numeracy Interventions (Aunio, in press) (4)

- Previous meta-analysis (Chodura et al. 2015; Dennis et al. 2016; Jitendra et al. 2018; Mononen et al. 2012) reported weaknesses:
  - Longitudinal effects of interventions not measured/not reported
  - Identification of children with mathematical difficulties: challenges in selection and outcome measures, cut-off criteria

Mini review in the beginning of the year 2018
- studies published 2014-2017
- 7 studies (small group, quasi-experimental with control group)
- US
- 3/7 used some kind of randomization, supplementary, children with low performance
- 3 studies: also low income background (Barnes et al 2016; Dyson et al. 2015; Hassinger-Das, Jordan & Dyson 2015)
Recent Development with Early Numeracy Interventions (Aunio, in press) (5)

- ROOTS was used in 3/7 (Clarke, Doabler, Smolkowski, Baker, Fien & Cary, 2016; Clarke, Doabler, Smolkowski, Kurtz-Nelson, Fien, Baker & Kosty 2016; Doabler, Clarke, Kosty, Kurtz-Nelson, Fien, Baker & Kosty, 2016)

- All 7 studies used standardized measurements (Number Sense Brief Screener, SAT; TEMA-3, WJ-III) but also measurements designed by research group were used (EN-CBM, ASPENS)

- All 7 studies report significant intervention effects
  - But only 3 reported delayed measurements (lasting effects) (Clarke, Doabler, Smolkowski, Kurtz-Nelson, Fien, Baker & Kosty; 2016; Dyson, Jordan, Baliakoff & Hassinger-Das 2015; Hassinger-Das, Jordan & Dyson 2015)

- Explicit and systematic small group interventions have effects on early numeracy learning of low performing students.
<table>
<thead>
<tr>
<th>Program</th>
<th>Age group</th>
<th>Skills practiced</th>
<th>Authors/source</th>
</tr>
</thead>
<tbody>
<tr>
<td>NalleMatikka (Fin/Swe)</td>
<td>prekindergarten</td>
<td>Math concepts, math phenomena in every day environment</td>
<td>Mattinen, Räsänen, Hannula &amp; Lehtinen (2010) Niilo Mäki Institute</td>
</tr>
<tr>
<td>Minäkin lasken!</td>
<td>4-7 years</td>
<td>Numerical relational skills, counting skills</td>
<td>Van Luit, Aunio &amp; Räsänen (2010) Niilo Mäki Institute</td>
</tr>
<tr>
<td>Selkis -yhteenlasku</td>
<td>grades 1-6</td>
<td>Addition skills</td>
<td>Koponen, Mononen, Kumpulainen &amp; Puura,(2011) Niilo Mäki Institute</td>
</tr>
<tr>
<td>Selkis-vähennyslasku</td>
<td>grades 1-6</td>
<td>Subtraction skills</td>
<td>Koponen, Mononen &amp; Latva ( 2011) Niilo Mäki Institute</td>
</tr>
<tr>
<td>HoPE – hopeisen pöllön etsintä Salaisten Lukujen valtakunnassa</td>
<td>grades 4-6</td>
<td>Word problem solving skills</td>
<td>Vauras &amp; Kinnunen (2003) Turun yliopisto, Oppimistutkimuksen keskus.</td>
</tr>
</tbody>
</table>
# Evidence-based programs (free for users)

<table>
<thead>
<tr>
<th>Program</th>
<th>Age group</th>
<th>Skills practiced</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerorata (Fin/Swe)</td>
<td>5-8 years</td>
<td>Number sense, counting skills, arithmetic skills</td>
<td><a href="http://www.lukimat.fi">www.lukimat.fi</a> and <a href="http://www.thenumberrace.com">www.thenumberrace.com</a></td>
</tr>
<tr>
<td>EkapeliMatikka (Fin)</td>
<td>5-8 years</td>
<td>Number knowledge, counting skills, addition skills</td>
<td><a href="http://www.lukimat.fi">www.lukimat.fi</a></td>
</tr>
<tr>
<td>Vektor (Fin/Swe)</td>
<td>5-10 years (?)</td>
<td>Number knowledge, addition and subtraction skills, visuospatial skills</td>
<td><a href="http://cognitionmatters.org(fi)">http://cognitionmatters.org(fi)</a></td>
</tr>
<tr>
<td>ThinkMath (Fin/Engl)</td>
<td>5-8 years</td>
<td>Numerical relational skills, counting skills, arithmetic skills</td>
<td><a href="http://thinkmathglobal.com">thinkmathglobal.com</a></td>
</tr>
</tbody>
</table>
Produce evidence-based **mathematics and thinking skills** intervention materials for at-risk children in K-2 Grades

Materials will be provided for teachers via web service (free-of-cost) http://blogs.helsinki.fi/thinkmath/ or https://thinkmathglobal.com

Funded by the Finnish Ministry of Education and Culture (2011-2016)

Math intervention materials:

- Explicit teaching
- Several ways to practice
- Teacher led small groups (4-6 children)
- Twice a week 45 minutes 8 weeks
- Counting skills, numerical relational skills, basic arithmetic skills

Ministry of Education and Culture is still active…. 
Adolescents
LEARNING DIFFICULTIES, PERCEIVED LEARNING DIFFICULTIES AND EDUCATIONAL DROP OUT
Aim: We were interested in what kind of performance and well-being profiles would be found in grade 9 students and how these profiles are related to dropout

Statistical analyses: Latent profile analysis, chi-square
To sum up: Why evidence based tools are important?

- Understanding the learning of mathematical core skills
  - with research knowledge we can understand and make estimations (e.g. educational interventions)

- Proper identification of those children who have difficulties in learning
  - Accurate measurements rather than lottery (Get rid of bad measurements)
  - Economically (Society) and individually important

- Intervention
  - Targets the most important skills,
  - Most efficient ways to teach and practice

- Prevent educational drop-out
Let’s collaborate and make the world better place for all children - no need for extra suffering with math learning

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